4.6-1 Resuelve las siguientes integrales indefinidas utilizando la propiedad de linealidad y la tabla de integrales inmediatas:

a) $\int (x-2)^2 \, dx$

b) $\int \dfrac{1}{x^4} \, dx$

c) $\int \dfrac{x^2-x+5}{x} \, dx$

d) $\int (3e^x - \operatorname{sen}x) \, dx$

e) $\int \sqrt[3]{x^2} \, dx$

f) $\int \dfrac{3}{5x^2+5} \, dx$

g) $\int \sqrt{\dfrac{4}{9-9x^2}} \, dx$

Solución

a) $\displaystyle\int (x-2)^2 \, dx = \int (x^2 - 4x + 4)\, dx = \int x^2 dx - 4\int x\,dx + 4\int dx =$

$$= \frac{x^3}{3} - 4\frac{x^2}{2} + 4x + C = \frac{x^3}{3} - 2x^2 + 4x + C$$

b) $\displaystyle\int \frac{1}{x^4} dx = \int x^{-4} dx = \frac{x^{-3}}{-3} + C = -\frac{1}{3}\frac{1}{x^3} + C$

c) $\displaystyle\int \frac{x^2-x+5}{x} dx = \int \left(x - 1 + \frac{5}{x} \right) dx = \int x\,dx - \int dx + 5\int \frac{1}{x} dx =$

$$= \frac{x^2}{2} - x + 5\ln|x| + C$$

d) $\displaystyle\int (3e^x - \operatorname{sen}x)\,dx = 3\int e^x dx - \int \operatorname{sen}x\,dx = 3e^x + \cos x + C$

e) $\displaystyle\int \sqrt[3]{x^2} dx = \int x^{2/3} dx = \frac{x^{5/3}}{5/3} + C = \frac{3}{5}x^{5/3} + C = \frac{3}{5}\sqrt[3]{x^5} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$

f) $\displaystyle\int \frac{3}{5x^2+5} dx = \frac{3}{5}\int \frac{1}{x^2+1} dx = \frac{3}{5}\operatorname{arctag}x + C$

g) $\displaystyle\int \sqrt{\frac{4}{9-9x^2}} dx = \int \sqrt{\frac{4}{9}}\sqrt{\frac{1}{1-x^2}} dx = \sqrt{\frac{4}{9}}\int \frac{1}{\sqrt{1-x^2}} dx = \frac{2}{3}\operatorname{arcsen}x + C$

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Ejercicios resueltos 1

4.6-2 Resuelve las siguientes integrales indefinidas utilizando algún cambio de variable apropiado:

a) $\displaystyle\int x\sqrt{x-1}\,dx$

b) $\displaystyle\int \frac{senx}{\sqrt{\cos x}}\,dx$

c) $\displaystyle\int \frac{x^2}{x^3-2}\,dx$

d) $\displaystyle\int \left(e^x-3\right)^4 e^x dx$

e) $\displaystyle\int \frac{2x}{1+x^4}\,dx$

f) $\displaystyle\int \frac{\ln x}{x}\,dx$

g) $\displaystyle\int \frac{e^{tagx}}{\cos^2 x}\,dx$

Solución

a) $\displaystyle\int x\sqrt{x-1}\,dx = \begin{pmatrix} t=x-1 \\ dt=dx \end{pmatrix} = \int (t+1)\sqrt{t}\,dt = \int t\sqrt{t}\,dt + \int \sqrt{t}\,dt =$

$$= \int t\cdot t^{1/2}\,dt + \int t^{1/2}\,dt = \int t^{3/2}\,dt + \int t^{1/2}\,dt = \frac{t^{5/2}}{5/2} + \frac{t^{3/2}}{3/2} + C =$$

$$= \frac{2}{5}t^{5/2} + \frac{2}{3}t^{3/2} + C = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

b) $\displaystyle\int \frac{senx}{\sqrt{\cos x}}\,dx = \begin{pmatrix} t=\cos x \\ dt=-senxdx \end{pmatrix} = \int \frac{-dt}{\sqrt{t}} = -\int \frac{1}{t^{1/2}}\,dt = -\int t^{-1/2}\,dt =$

$$= -\frac{t^{1/2}}{1/2} + C = -2t^{1/2} + C = -2\sqrt{t} + C = -2\sqrt{\cos x} + C$$

c) $\displaystyle\int \frac{x^2}{x^3-2}\,dx = \begin{pmatrix} t=x^3-2 \\ dt=3x^2dx \end{pmatrix} = \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln|t| + C = \frac{1}{3}\ln|x^3-2| + C$

d) $\displaystyle\int \left(e^x-3\right)^4 e^x dx = \begin{pmatrix} t=e^x-3 \\ dt=e^x dx \end{pmatrix} = \int t^4 dt = \frac{t^5}{5} + C = \frac{\left(e^x-3\right)^5}{5} + C$

e) $\displaystyle\int \frac{2x}{1+x^4}\,dx = \begin{pmatrix} t=x^2 \\ dt=2xdx \end{pmatrix} = \int \frac{1}{1+t^2}\,dt = arctagt + C = arctag\left(x^2\right) + C$

f) $\displaystyle\int \frac{\ln x}{x}\,dx = \begin{pmatrix} t=\ln x \\ dt=\dfrac{1}{x}\,dx \end{pmatrix} = \int t\,dt = \frac{t^2}{2} + C = \frac{\left(\ln|x|\right)^2}{2} + C$

g) $\displaystyle\int \frac{e^{tagx}}{\cos^2 x}\,dx = \begin{pmatrix} t=tagx \\ dt=\dfrac{1}{\cos^2 x}\,dx \end{pmatrix} = \int e^t dt = e^t + C = e^{tagx} + C$

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Ejercicios resueltos 2

4.6-3 Resuelve las siguientes integrales indefinidas utilizando el método de integración por partes:

$a)$ $\int \ln x\, dx$

$b)$ $\int x \ln x\, dx$

$c)$ $\int xe^x dx$

$d)$ $\int x^2 e^x dx$

$e)$ $\int arcsen x\, dx$

$f)$ $\int e^x sen x\, dx$

$g)$ $\int x^2 sen x\, dx$

Solución

$a)$ $\displaystyle\int \ln x\, dx = \begin{pmatrix} u = \ln x & \Rightarrow & du = \dfrac{dx}{x} \\ dv = dx & \Rightarrow & v = x \end{pmatrix} = x\ln x - \int x\dfrac{dx}{x} = x\ln x - \int dx =$

$\qquad = x\ln x - x + C$

$b)$ $\displaystyle\int x\ln x\, dx = \begin{pmatrix} u = \ln x & \Rightarrow & du = \dfrac{dx}{x} \\ dv = x\, dx & \Rightarrow & v = \dfrac{x^2}{2} \end{pmatrix} = \dfrac{x^2}{2}\ln x - \dfrac{1}{2}\int x^2 \dfrac{dx}{x} =$

$\qquad = \dfrac{x^2}{2}\ln x - \dfrac{1}{2}\int x\, dx = \dfrac{x^2}{2}\ln x - \dfrac{1}{2}\dfrac{x^2}{2} + C = \dfrac{x^2}{2}\ln x - \dfrac{x^2}{4} + C$

$c)$ $\displaystyle\int xe^x dx = \begin{pmatrix} u = x & \Rightarrow & du = dx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = xe^x - \int e^x dx = xe^x - e^x + C$

$d)$ $\displaystyle\int x^2 e^x dx = \begin{pmatrix} u = x^2 & \Rightarrow & du = 2x\, dx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = x^2 e^x - 2\int xe^x dx =$

$\qquad = \begin{pmatrix} u = x & \Rightarrow & du = dx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = x^2 e^x - 2\left[xe^x - \int e^x dx \right] =$

$\qquad = x^2 e^x - 2xe^x + 2\int e^x dx = x^2 e^x - 2xe^x + 2e^x + C$

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Ejercicios resueltos 3

$$e)\int arcsenx\,dx = \begin{pmatrix} u = arcsenx & \Rightarrow & du = \dfrac{dx}{\sqrt{1-x^2}} \\ dv = dx & \Rightarrow & v = x \end{pmatrix} = xarcsenx - \int \dfrac{x}{\sqrt{1-x^2}}dx =$$

$$= xarcsenx - \int x\left(1-x^2\right)^{-1/2}dx = \begin{pmatrix} t = 1-x^2 \\ dt = -2x\,dx \end{pmatrix} =$$

$$= xarcsenx + \dfrac{1}{2}\int t^{-1/2}dt = xarcsenx + \dfrac{1}{2}\dfrac{t^{1/2}}{1/2} + C =$$

$$= xarcsenx + \left(1-x^2\right)^{1/2} + C = xarcsenx + \sqrt{1-x^2} + C$$

$$f)\int e^x senx\,dx = \begin{pmatrix} u = e^x & \Rightarrow & du = e^x dx \\ dv = senx\,dx & \Rightarrow & v = -\cos x \end{pmatrix} = -e^x \cos x + \int e^x \cos x\,dx =$$

$$= \begin{pmatrix} u = e^x & \Rightarrow & du = e^x dx \\ dv = \cos x\,dx & \Rightarrow & v = senx \end{pmatrix} =$$

$$= -e^x \cos x + e^x senx - \int e^x senx\,dx$$

$$\Rightarrow 2\int e^x senx\,dx = -e^x \cos x + e^x senx$$

$$\Rightarrow \int e^x senx\,dx = \dfrac{e^x\left(senx - \cos x\right)}{2} + C$$

$$g)\int x^2 senx\,dx = \begin{pmatrix} u = x^2 & \Rightarrow & du = 2x\,dx \\ dv = senx\,dx & \Rightarrow & v = -\cos x \end{pmatrix} = -x^2 \cos x + 2\int x\cos x\,dx =$$

$$= \begin{pmatrix} u = x & \Rightarrow & du = dx \\ dv = \cos x\,dx & \Rightarrow & v = senx \end{pmatrix} =$$

$$= -x^2 \cos x + 2xsenx - 2\int senx\,dx =$$

$$= -x^2 \cos x + 2xsenx + 2\cos x + C$$

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Ejercicios resueltos 4

4.6-4 Resuelve las siguientes integrales indefinidas de funciones racionales:

a) $\displaystyle\int \frac{x^2+1}{x^3+x^2-2x}dx$

b) $\displaystyle\int \frac{x^2+1}{x^3-6x^2+12x-8}dx$

c) $\displaystyle\int \frac{4}{x^3-5x^2+7x-3}dx$

d) $\displaystyle\int \frac{x^2-3x+2}{x^2-9}dx$

e) $\displaystyle\int \frac{x^2+x-8}{x^3-4x^2}dx$

f) $\displaystyle\int \frac{2x^2-x+1}{x^2+8x+16}dx$

g) $\displaystyle\int \frac{x^4+3x^3-2x^2+1}{x+5}dx$

Solución

a) $\displaystyle\int \frac{x^2+1}{x^3+x^2-2x}dx \Rightarrow x^3+x^2-2x=x\left(x^2+x-2\right)=x\left(x-1\right)\left(x+2\right)$

$$\frac{x^2+1}{x^3+x^2-2x}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+2}$$

$$x^2+1=A\left(x-1\right)\left(x+2\right)+Bx\left(x+2\right)+Cx\left(x-1\right)$$

$$\left.\begin{array}{lllll} x=&0:&1&=&-2A \\ x=&1:&2&=&3B \\ x=&-2:&5&=&6C \end{array}\right\} \Rightarrow \begin{array}{l} A=-1/2 \\ B=2/3 \\ C=5/6 \end{array}$$

$$\int \frac{x^2+1}{x^3+x^2-2x}dx=-\frac{1}{2}\int\frac{dx}{x}+\frac{2}{3}\int\frac{dx}{x-1}+\frac{5}{6}\int\frac{dx}{x+2}=$$

$$=-\frac{1}{2}\ln|x|+\frac{2}{3}\ln|x-1|+\frac{5}{6}\ln|x+2|+C$$

b) $\displaystyle\int \frac{x^2+1}{x^3-6x^2+12x-8}dx \Rightarrow x^3-6x^2+12x-8=\left(x-2\right)^3$

$$\frac{x^2+1}{\left(x-2\right)^3}=\frac{A}{x-2}+\frac{B}{\left(x-2\right)^2}+\frac{C}{\left(x-2\right)^3}$$

$$x^2+1=A\left(x-2\right)^2+B\left(x-2\right)+C=Ax^2-4Ax+4A+Bx-2B+C$$

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Ejercicios resueltos 5

$$\begin{array}{rcll}
x^2: & 1 & = & A \\
x^1: & 0 & = & -4A+B \\
x^0: & 1 & = & 4A-2B+C
\end{array}\left.\right\}\Rightarrow \begin{array}{l} A=1 \\ B=4 \\ C=5 \end{array}$$

$$\int \frac{x^2+1}{x^3-6x^2+12x-8}\,dx = \int \frac{dx}{x-2} + 4\int \frac{dx}{(x-2)^2} + 5\int \frac{dx}{(x-2)^3} =$$

$$= \ln|x-2| - \frac{4}{x-2} - \frac{5}{2(x-2)^2} + C$$

c) $\displaystyle\int \frac{4}{x^3-5x^2+7x-3}\,dx \Rightarrow x^3-5x^2+7x-3=(x-1)^2(x-3)$

$$\frac{4}{(x-1)^2(x-3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-3}$$

$$4 = A(x-1)(x-3) + B(x-3) + C(x-1)^2$$

$$\begin{array}{rcll}
x=1: & 4 & = & -2B \\
x=3: & 4 & = & 4C \\
x=0: & 4 & = & 3A-3B+C
\end{array}\left.\right\}\Rightarrow \begin{array}{l} A=-1 \\ B=-2 \\ C=1 \end{array}$$

$$\int \frac{4}{x^3-5x^2+7x-3}\,dx = -\int \frac{dx}{x-1} - 2\int \frac{dx}{(x-1)^2} + \int \frac{dx}{x-3} =$$

$$= -\ln|x-1| + \frac{2}{x-1} + \ln|x-3| + C$$

d) $\displaystyle\int \frac{x^2-3x+2}{x^2-9}\,dx \Rightarrow x^2-3x+2 = 1\cdot(x^2-9) + (-3x+11)$

$$\int \frac{x^2-3x+2}{x^2-9}\,dx = \int dx + \int \frac{-3x+11}{x^2-9}\,dx \Rightarrow x^2-9 = (x-3)(x+3)$$

$$\frac{-3x+11}{x^2-9} = \frac{A}{x-3} + \frac{B}{x+3} \Rightarrow -3x+11 = A(x+3) + B(x-3)$$

$$\begin{array}{rcll}
x= 3: & 2 & = & 6A \\
x=-3: & 20 & = & -6B
\end{array}\left.\right\}\Rightarrow \begin{array}{l} A=1/3 \\ B=-10/3 \end{array}$$

$$\int \frac{x^2-3x+2}{x^2-9}\,dx = \int dx + \int \frac{-3x+11}{x^2-9}\,dx = x + \frac{1}{3}\int \frac{dx}{x-3} - \frac{10}{3}\int \frac{dx}{x+3} =$$

$$= x + \frac{1}{3}\ln|x-3| - \frac{10}{3}\ln|x+3| + C$$

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Ejercicios resueltos 6

$e)$ $\displaystyle\int \frac{x^2 + x - 8}{x^3 - 4x^2}dx \Rightarrow x^3 - 4x^2 = x^2(x - 4)$

$$\frac{x^2 + x - 8}{x^3 - 4x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 4}$$

$$x^2 + x - 8 = Ax(x - 4) + B(x - 4) + Cx^2 = Ax^2 - 4Ax + Bx - 4B + Cx^2$$

$$\begin{array}{rcll} x^2: & 1 & = & A + C \\ x^1: & 1 & = & -4A + B \\ x^0: & -8 & = & -4B \end{array}\Bigg\} \Rightarrow \begin{array}{l} A = 1/4 \\ B = 2 \\ C = 3/4 \end{array}$$

$$\int \frac{x^2 + x - 8}{x^3 - 4x^2}dx = \frac{1}{4}\int \frac{1}{x}dx + 2\int \frac{1}{x^2}dx + \frac{3}{4}\int \frac{1}{(x - 4)}dx =$$

$$= \frac{1}{4}\ln|x| - \frac{2}{x} + \frac{3}{4}\ln|x - 4| + C$$

$f)$ $\displaystyle\int \frac{2x^2 - x + 1}{x^2 + 8x + 16}dx \Rightarrow 2x^2 - x + 1 = 2\cdot(x^2 + 8x + 16) + (-17x - 31)$

$$\int \frac{2x^2 - x + 1}{x^2 + 8x + 16}dx = 2\int dx + \int \frac{-17x - 31}{x^2 + 8x + 16}dx \Rightarrow x^2 + 8x + 16 = (x + 4)^2$$

$$\frac{-17x - 31}{(x + 4)^2} = \frac{A}{x + 4} + \frac{B}{(x + 4)^2} \Rightarrow -17x - 31 = A(x + 4) + B$$

$$\begin{array}{rcll} x^1: & -17 & = & A \\ x^0: & -31 & = & 4A + B \end{array}\Bigg\} \Rightarrow \begin{array}{l} A = -17 \\ B = 37 \end{array}$$

$$\int \frac{2x^2 - x + 1}{x^2 + 8x + 16}dx = 2\int dx - 17\int \frac{1}{x + 4}dx + 37\int \frac{1}{(x + 4)^2}dx =$$

$$= 2x - 17\ln|x + 4| - \frac{37}{x + 4} + C$$

$g)$ $\displaystyle\int \frac{x^4 + 3x^3 - 2x^2 + 1}{x + 5}dx$

$$x^4 + 3x^3 - 2x^2 + 1 = (x^3 - 2x^2 + 8x - 40)\cdot(x + 5) + 201$$

$$\int \frac{x^4 + 3x^3 - 2x^2 + 1}{x + 5}dx = \int (x^3 - 2x^2 + 8x - 40)dx + \int \frac{201}{x + 5}dx =$$

$$= \frac{x^4}{4} - 2\frac{x^3}{3} + 8\frac{x^2}{2} - 40x + 201\ln|x + 5| + C$$

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