# Ejercicios resueltos

**1** Resuelve las siguientes integrales indefinidas utilizando la propiedad de linealidad y la tabla de integrales inmediatas:

$$a)\ \int (x-2)^2\,dx$$

$$b)\ \int \frac{1}{x^4}\,dx$$

$$c)\ \int \frac{x^2-x+5}{x}\,dx$$

$$d)\ \int \left(3e^x - senx\right)dx$$

$$e)\ \int \sqrt[3]{x^2}\,dx$$

$$f)\ \int \frac{3}{5x^2+5}\,dx$$

$$g)\ \int \sqrt{\frac{4}{9-9x^2}}\,dx$$

***Solución***

$$a)\ \int (x-2)^2\,dx = \int \left(x^2-4x+4\right)dx = \int x^2dx - 4\int xdx + 4\int dx =$$

$$= \frac{x^3}{3} - 4\frac{x^2}{2} + 4x + C = \frac{x^3}{3} - 2x^2 + 4x + C$$

$$b)\ \int \frac{1}{x^4}\,dx = \int x^{-4}dx = \frac{x^{-3}}{-3} + C = -\frac{1}{3}\frac{1}{x^3} + C$$

$$c)\ \int \frac{x^2-x+5}{x}\,dx = \int \left(x - 1 + \frac{5}{x}\right)dx = \int xdx - \int dx + 5\int \frac{1}{x}dx =$$

$$= \frac{x^2}{2} - x + 5\ln|x| + C$$

$$d)\ \int \left(3e^x - senx\right)dx = 3\int e^x dx - \int senxdx = 3e^x + \cos x + C$$

$$e)\ \int \sqrt[3]{x^2}\,dx = \int x^{2/3}dx = \frac{x^{5/3}}{5/3} + C = \frac{3}{5}x^{5/3} + C = \frac{3}{5}\sqrt[3]{x^5} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$$

$$f)\ \int \frac{3}{5x^2+5}\,dx = \frac{3}{5}\int \frac{1}{x^2+1}\,dx = \frac{3}{5}arctagx + C$$

$$g)\ \int \sqrt{\frac{4}{9-9x^2}}\,dx = \int \sqrt{\frac{4}{9}}\sqrt{\frac{1}{1-x^2}}\,dx = \sqrt{\frac{4}{9}}\int \frac{1}{\sqrt{1-x^2}}\,dx = \frac{2}{3}arcsenx + C$$

**2** Resuelve las siguientes integrales indefinidas utilizando algún cambio de variable apropiado:

$$a)\ \int x\sqrt{x-1}\,dx$$

$$b)\ \int \frac{senx}{\sqrt{\cos x}}\,dx$$

$$c)\ \int \frac{x^2}{x^3-2}\,dx$$

$$d)\ \int \left(e^x-3\right)^4 e^x dx$$

$$e)\ \int \frac{2x}{1+x^4}\,dx$$

$$f)\ \int \frac{\ln x}{x}\,dx$$

$$g)\ \int \frac{e^{tagx}}{\cos^2 x}\,dx$$

**Solución**

$$a)\ \int x\sqrt{x-1}\,dx = \begin{pmatrix} t = x-1 \\ dt = dx \end{pmatrix} = \int (t+1)\sqrt{t}\,dt = \int t\sqrt{t}\,dt + \int \sqrt{t}\,dt =$$

$$= \int t\cdot t^{1/2}dt + \int t^{1/2}dt = \int t^{3/2}dt + \int t^{1/2}dt = \frac{t^{5/2}}{5/2} + \frac{t^{3/2}}{3/2} + C =$$

$$= \frac{2}{5}t^{5/2} + \frac{2}{3}t^{3/2} + C = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

$$b)\ \int \frac{senx}{\sqrt{\cos x}}\,dx = \begin{pmatrix} t = \cos x \\ dt = -senxdx \end{pmatrix} = \int \frac{-dt}{\sqrt{t}} = -\int \frac{1}{t^{1/2}}\,dt = -\int t^{-1/2}dt =$$

$$= -\frac{t^{1/2}}{1/2} + C = -2t^{1/2} + C = -2\sqrt{t} + C = -2\sqrt{\cos x} + C$$

$$c)\ \int \frac{x^2}{x^3-2}\,dx = \begin{pmatrix} t = x^3-2 \\ dt = 3x^2dx \end{pmatrix} = \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln|t| + C = \frac{1}{3}\ln\left|x^3-2\right| + C$$

$$d)\ \int \left(e^x-3\right)^4 e^x dx = \begin{pmatrix} t = e^x-3 \\ dt = e^x dx \end{pmatrix} = \int t^4 dt = \frac{t^5}{5} + C = \frac{\left(e^x-3\right)^5}{5} + C$$

$$e)\ \int \frac{2x}{1+x^4}\,dx = \begin{pmatrix} t = x^2 \\ dt = 2xdx \end{pmatrix} = \int \frac{1}{1+t^2}\,dt = arctagt + C = arctag\left(x^2\right) + C$$

$$f)\ \int \frac{\ln x}{x}\,dx = \begin{pmatrix} t = \ln x \\ dt = \frac{1}{x}dx \end{pmatrix} = \int t\,dt = \frac{t^2}{2} + C = \frac{\left(\ln|x|\right)^2}{2} + C$$

$$g)\ \int \frac{e^{tagx}}{\cos^2 x}\,dx = \begin{pmatrix} t = tagx \\ dt = \frac{1}{\cos^2 x}dx \end{pmatrix} = \int e^t dt = e^t + C = e^{tagx} + C$$

**3** Resuelve las siguientes integrales indefinidas utilizando el método de integración por partes:

$$a)\ \int \ln x dx$$

$$b)\ \int x \ln x dx$$

$$c)\ \int x e^x dx$$

$$d)\ \int x^2 e^x dx$$

$$e)\ \int arcsenx dx$$

$$f)\ \int e^x senx dx$$

$$g)\ \int x^2 senx dx$$

***Solución***

$$a)\ \int \ln x dx = \begin{pmatrix} u = \ln x & \Rightarrow & du = \dfrac{dx}{x} \\ dv = dx & \Rightarrow & v = x \end{pmatrix} = x\ln x - \int x \frac{dx}{x} = x \ln x - \int dx =$$

$$= x\ln x - x + C$$

$$b)\ \int x \ln x dx = \begin{pmatrix} u = \ln x & \Rightarrow & du = \dfrac{dx}{x} \\ dv = xdx & \Rightarrow & v = \dfrac{x^2}{2} \end{pmatrix} = \frac{x^2}{2}\ln x - \frac{1}{2}\int x^2 \frac{dx}{x} =$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int xdx = \frac{x^2}{2}\ln x - \frac{1}{2}\frac{x^2}{2} + C = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$$c)\ \int xe^x dx = \begin{pmatrix} u = x & \Rightarrow & du = dx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = xe^x - \int e^x dx = xe^x - e^x + C$$

$$d)\ \int x^2 e^x dx = \begin{pmatrix} u = x^2 & \Rightarrow & du = 2xdx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = x^2 e^x - 2\int xe^x dx =$$

$$= \begin{pmatrix} u = x & \Rightarrow & du = dx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = x^2 e^x - 2\left[ xe^x - \int e^x dx \right] =$$

$$= x^2 e^x - 2xe^x + 2\int e^x dx = x^2 e^x - 2xe^x + 2e^x + C$$

$$e)\int arcsenx\,dx=\begin{pmatrix} u=arcsenx & \Rightarrow & du=\dfrac{dx}{\sqrt{1-x^2}} \\ dv=dx & \Rightarrow & v=x \end{pmatrix}=xarcsenx-\int\frac{x}{\sqrt{1-x^2}}dx=$$

$$=xarcsenx-\int x\left(1-x^2\right)^{-1/2}dx=\begin{pmatrix} t=1-x^2 \\ dt=-2xdx \end{pmatrix}=$$

$$=xarcsenx+\frac{1}{2}\int t^{-1/2}dt=xarcsenx+\frac{1}{2}\frac{t^{1/2}}{1/2}+C=$$

$$=xarcsenx+\left(1-x^2\right)^{1/2}+C=xarcsenx+\sqrt{1-x^2}+C$$

$$f)\int e^x senx\,dx=\begin{pmatrix} u=e^x & \Rightarrow & du=e^x dx \\ dv=senxdx & \Rightarrow & v=-\cos x \end{pmatrix}=-e^x\cos x+\int e^x\cos xdx=$$

$$=\begin{pmatrix} u=e^x & \Rightarrow & du=e^x dx \\ dv=\cos xdx & \Rightarrow & v=senx \end{pmatrix}=$$

$$=-e^x\cos x+e^x senx-\int e^x senxdx$$

$$\Rightarrow 2\int e^x senxdx=-e^x\cos x+e^x senx$$

$$\Rightarrow \int e^x senxdx=\frac{e^x\left(senx-\cos x\right)}{2}+C$$

$$g)\int x^2 senx\,dx=\begin{pmatrix} u=x^2 & \Rightarrow & du=2xdx \\ dv=senxdx & \Rightarrow & v=-\cos x \end{pmatrix}=-x^2\cos x+2\int x\cos xdx=$$

$$=\begin{pmatrix} u=x & \Rightarrow & du=dx \\ dv=\cos xdx & \Rightarrow & v=senx \end{pmatrix}=$$

$$=-x^2\cos x+2xsenx-2\int senxdx=$$

$$=-x^2\cos x+2xsenx+2\cos x+C$$

4 Resuelve las siguientes integrales indefinidas de funciones racionales:

$$a)\ \int \frac{x^2+1}{x^3+x^2-2x}dx$$

$$b)\ \int \frac{x^2+1}{x^3-6x^2+12x-8}dx$$

$$c)\ \int \frac{4}{x^3-5x^2+7x-3}dx$$

$$d)\ \int \frac{x^2-3x+2}{x^2-9}dx$$

$$e)\ \int \frac{x^2+x-8}{x^3-4x^2}dx$$

$$f)\ \int \frac{2x^2-x+1}{x^2+8x+16}dx$$

$$g)\ \int \frac{x^4+3x^3-2x^2+1}{x+5}dx$$

**Solución**

$$a)\ \int \frac{x^2+1}{x^3+x^2-2x}dx \Rightarrow x^3+x^2-2x = x\left(x^2+x-2\right) = x(x-1)(x+2)$$

$$\frac{x^2+1}{x^3+x^2-2x} = \frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+2}$$

$$x^2+1 = A(x-1)(x+2)+Bx(x+2)+Cx(x-1)$$

$$\left.\begin{array}{rcl} x=0: & 1 = & -2A \\ x=1: & 2 = & 3B \\ x=-2: & 5 = & 6C \end{array}\right\} \Rightarrow \begin{array}{l} A=-1/2 \\ B=2/3 \\ C=5/6 \end{array}$$

$$\int \frac{x^2+1}{x^3+x^2-2x}dx = -\frac{1}{2}\int\frac{dx}{x}+\frac{2}{3}\int\frac{dx}{x-1}+\frac{5}{6}\int\frac{dx}{x+2} =$$

$$= -\frac{1}{2}\ln|x|+\frac{2}{3}\ln|x-1|+\frac{5}{6}\ln|x+2|+C$$

$$b)\ \int \frac{x^2+1}{x^3-6x^2+12x-8}dx \Rightarrow x^3-6x^2+12x-8 = (x-2)^3$$

$$\frac{x^2+1}{(x-2)^3} = \frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}$$

$$x^2+1 = A(x-2)^2+B(x-2)+C = Ax^2-4Ax+4A+Bx-2B+C$$

$$\left.\begin{array}{lcc} x^2: & 1 & = & A \\ x^1: & 0 & = & -4A+B \\ x^0: & 1 & = & 4A-2B+C \end{array}\right\} \Rightarrow \begin{array}{l} A=1 \\ B=4 \\ C=5 \end{array}$$

$$\int \frac{x^2+1}{x^3-6x^2+12x-8}dx = \int \frac{dx}{x-2} + 4\int \frac{dx}{(x-2)^2} + 5\int \frac{dx}{(x-2)^3} =$$

$$= \ln|x-2| - \frac{4}{x-2} - \frac{5}{2(x-2)^2} + C$$

*c*) $\int \frac{4}{x^3-5x^2+7x-3}dx \Rightarrow x^3-5x^2+7x-3=(x-1)^2(x-3)$

$$\frac{4}{(x-1)^2(x-3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-3}$$

$$4 = A(x-1)(x-3) + B(x-3) + C(x-1)^2$$

$$\left.\begin{array}{lcc} x=1: & 4 & = & -2B \\ x=3: & 4 & = & 4C \\ x=0: & 4 & = & 3A-3B+C \end{array}\right\} \Rightarrow \begin{array}{l} A=-1 \\ B=-2 \\ C=1 \end{array}$$

$$\int \frac{4}{x^3-5x^2+7x-3}dx = -\int \frac{dx}{x-1} - 2\int \frac{dx}{(x-1)^2} + \int \frac{dx}{x-3} =$$

$$= -\ln|x-1| + \frac{2}{x-1} + \ln|x-3| + C$$

*d*) $\int \frac{x^2-3x+2}{x^2-9}dx \Rightarrow x^2-3x+2 = 1\cdot(x^2-9)+(-3x+11)$

$$\int \frac{x^2-3x+2}{x^2-9}dx = \int dx + \int \frac{-3x+11}{x^2-9}dx \Rightarrow x^2-9=(x-3)(x+3)$$

$$\frac{-3x+11}{x^2-9} = \frac{A}{x-3} + \frac{B}{x+3} \Rightarrow -3x+11 = A(x+3)+B(x-3)$$

$$\left.\begin{array}{lcc} x=\ \ 3: & 2 & = & 6A \\ x=-3: & 20 & = & -6B \end{array}\right\} \Rightarrow \begin{array}{l} A=1/3 \\ B=-10/3 \end{array}$$

$$\int \frac{x^2-3x+2}{x^2-9}dx = \int dx + \int \frac{-3x+11}{x^2-9}dx = x + \frac{1}{3}\int \frac{dx}{x-3} - \frac{10}{3}\int \frac{dx}{x+3} =$$

$$= x + \frac{1}{3}\ln|x-3| - \frac{10}{3}\ln|x+3| + C$$

e) $\int \frac{x^2+x-8}{x^3-4x^2}dx \Rightarrow x^3-4x^2=x^2(x-4)$

$$\frac{x^2+x-8}{x^3-4x^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-4}$$

$$x^2+x-8=Ax(x-4)+B(x-4)+Cx^2=Ax^2-4Ax+Bx-4B+Cx^2$$

$$\left.\begin{array}{lcc} x^2: & 1 & = & A+C \\ x^1: & 1 & = & -4A+B \\ x^0: & -8 & = & -4B \end{array}\right\} \Rightarrow \begin{array}{l} A=1/4 \\ B=2 \\ C=3/4 \end{array}$$

$$\int \frac{x^2+x-8}{x^3-4x^2}dx=\frac{1}{4}\int\frac{1}{x}dx+2\int\frac{1}{x^2}dx+\frac{3}{4}\int\frac{1}{(x-4)}dx=$$

$$=\frac{1}{4}\ln|x|-\frac{2}{x}+\frac{3}{4}\ln|x-4|+C$$

f) $\int \frac{2x^2-x+1}{x^2+8x+16}dx \Rightarrow 2x^2-x+1=2\cdot(x^2+8x+16)+(-17x-31)$

$$\int \frac{2x^2-x+1}{x^2+8x+16}dx=2\int dx+\int\frac{-17x-31}{x^2+8x+16}dx \Rightarrow x^2+8x+16=(x+4)^2$$

$$\frac{-17x-31}{(x+4)^2}=\frac{A}{x+4}+\frac{B}{(x+4)^2} \Rightarrow -17x-31=A(x+4)+B$$

$$\left.\begin{array}{lcc} x^1: & -17 & = & A \\ x^0: & -31 & = & 4A+B \end{array}\right\} \Rightarrow \begin{array}{l} A=-17 \\ B=37 \end{array}$$

$$\int \frac{2x^2-x+1}{x^2+8x+16}dx=2\int dx-17\int\frac{1}{x+4}dx+37\int\frac{1}{(x+4)^2}dx=$$

$$=2x-17\ln|x+4|-\frac{37}{x+4}+C$$

g) $\int \frac{x^4+3x^3-2x^2+1}{x+5}dx$

$$x^4+3x^3-2x^2+1=(x^3-2x^2+8x-40)\cdot(x+5)+201$$

$$\int \frac{x^4+3x^3-2x^2+1}{x+5}dx=\int(x^3-2x^2+8x-40)dx+\int\frac{201}{x+5}dx=$$

$$=\frac{x^4}{4}-2\frac{x^3}{3}+8\frac{x^2}{2}-40x+201\ln|x+5|+C$$