

Procedimientos de integración

Ejercicios propuestos

$$1) \int \frac{x^4 - x^3 - x - 1}{x^3 - x^2} dx = \frac{x^2}{2} + 2\text{Log}x - \frac{1}{x} - 2\text{Log}(x-1) + C$$

$$2) \int \frac{x^2 + 3x - 4}{x^2 - 2x - 8} dx = x + 4\text{Log}(x-4) + \text{Log}(x+2) + C$$

$$3) \int \frac{3x + 5}{x^3 - x^2 - x + 1} dx = \frac{1}{2}\text{Log}(x+1) - \frac{1}{2}\text{Log}(x-1) - \frac{4}{x-1} + C$$

$$4) \int \frac{8x^2 + 6x + 6}{x^3 - 3x^2 + 7x - 5} dx = \\ = 5\text{Log}(x-1) + \frac{3}{2}\text{Log}[(x-1)^2 + 4] + 11\text{arctg}\left(\frac{x-1}{2}\right) + C$$

$$5) \int \frac{3x-2}{x^2+x+1} dx = \frac{3}{2}\text{Log}\left[\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}\right] - \frac{7}{\sqrt{3}}\text{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

$$6) \int \frac{x^2 - 2}{x^3(x^2 + 1)^2} dx = \frac{1}{2} \frac{5x^2 + 2}{x^2(x^2 + 1)} + 5\text{Log}x - \frac{5}{2}\text{Log}(x^2 + 1) + C$$

$$7) \int \frac{x-2}{(x-1)(x^2+x+1)^2} dx = \frac{1}{3} \frac{3x+1}{x^2+x+1} - \frac{1}{9}\text{Log}(x-1) + \\ + \frac{1}{18}\text{Log}(x^2+x+1) + \frac{7\sqrt{3}}{9}\text{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

$$8) \int \frac{x^2 + 1}{(x-1)(x^2 + 2)^2} dx = \frac{1}{12} \frac{x-2}{x^2 + 2} + \frac{2}{9} \text{Log}(x-1) - \frac{1}{9} \text{Log}(x^2 + 2) - \frac{5\sqrt{2}}{72} \text{arctg}\left(\frac{x}{\sqrt{2}}\right) + C$$

$$9) \int \frac{dx}{x^2 - 9} = \frac{1}{6} \text{Log}(x-3) - \frac{1}{6} \text{Log}(x+3) + C$$

$$10) \int \frac{x}{x^2 - 3x - 4} dx = \frac{1}{5} \text{Log}(x+1) + \frac{4}{5} \text{Log}(x-4) + C$$

$$11) \int \frac{x^4}{(1-x)^3} dx = -\frac{x^2}{2} - 3x - 6\text{Log}(1-x) - \frac{4}{1-x} + \frac{1}{2} \frac{1}{(1-x)^2} + C$$

$$12) \int \frac{dx}{(x^2 - 1)^2} = -\frac{1}{4} \text{Log}(x-1) - \frac{1}{4} \frac{1}{x-1} + \frac{1}{4} \text{Log}(x+1) - \frac{1}{4} \frac{1}{x+1} + C$$

$$13) \int \frac{dx}{(x^2 + 1)(x+1)} = \frac{1}{2} \text{Log}(x+1) - \frac{1}{4} \text{Log}(x^2 + 1) + \frac{1}{2} \text{arctg} x + C$$

$$14) \int \frac{x^3}{x^4 + x^2 + 1} dx = \frac{1}{4} \text{Log}(x^4 + x^2 + 1) - \frac{\sqrt{3}}{6} \text{arctg}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) + C$$

$$15) \int \frac{dx}{(9 + x^2)^2} = \frac{1}{54} \text{arctg}\left(\frac{x}{3}\right) + \frac{x}{18(9 + x^2)} + C$$

$$16) \int \frac{2x^3 + x^2 + 4}{(4 + x^2)^2} dx = \frac{4}{x^2 + 4} + \text{Log}(x^2 + 4) + \frac{1}{2} \text{arctg}\left(\frac{x}{2}\right) + C$$

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$$17) \int \frac{dx}{x^3 - 1} = \frac{1}{3} \text{Log}(x-1) - \frac{1}{6} \text{Log}(x^2 + x + 1) - \frac{1}{\sqrt{3}} \text{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

$$18) \int \frac{dx}{(x^2 + x + 1)(x^2 + 1)} = \frac{1}{2} \text{Log}\left|\frac{x^2 + x + 1}{x^2 + 1}\right| + \frac{1}{\sqrt{3}} \text{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

$$19) \int \frac{3x^2 - 8x + 1}{(x-2)(x-3)(x+1)} dx = \text{Log}|x-2| + \text{Log}|x-3| + \text{Log}|x+1| + C$$

$$20) \int \frac{3x+1}{(x^2 - 1)^2} dx = \frac{1}{4} \text{Log}|x+1| + \frac{1}{2} \frac{1}{x+1} - \frac{1}{4} \text{Log}|x-1| - \frac{1}{x-1} + C$$

$$21) \int \frac{x^2 + 6x - 1}{(x-3)^2(x-1)} dx = -\frac{1}{2} \text{Log}|x-3| - \frac{13}{x-3} + \frac{3}{2} \text{Log}|x-1| + C$$

$$22) \int \frac{x^2 + x + 1}{x^2 - x + 1} dx = x + \text{Log}|x^2 - x + 1| + \frac{2}{\sqrt{3}} \text{arctg}\left(\frac{2x-1}{\sqrt{3}}\right) + C$$

$$23) \int \frac{dx}{(x^2 + 1)(x^2 - 1)} = -\frac{1}{4} \text{Log}|x+1| + \frac{1}{4} \text{Log}|x-1| - \frac{1}{2} \text{arctg}x + C$$

$$24) \int \frac{x^2 - 5x + 9}{x^2 - 5x + 6} dx = x - 3 \text{Log}|x-2| + 3 \text{Log}|x-3| + C$$

$$25) \int \frac{dx}{x^4 + 1} = \frac{1}{4\sqrt{2}} \text{Log}\left|\frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1}\right| + \\ + \frac{\sqrt{2}}{4} \text{arctg}(\sqrt{2}x + 1) + \frac{\sqrt{2}}{4} \text{arctg}(\sqrt{2}x - 1) + C$$

$$26) \int \frac{x^5}{(x^2 + 1)^3} dx = \frac{1}{2} \text{Log}(1 + x^2) + \frac{1}{1 + x^2} - \frac{1}{4} \frac{1}{(1 + x^2)^2} + C$$

$$27) \int \frac{dx}{x^4(x^3 + 1)^2} = -\frac{2}{3} \text{Log}|x^3| - \frac{1}{3} \frac{1}{x^3} + \frac{2}{3} \text{Log}|x^3 + 1| - \frac{1}{3} \frac{1}{x^3 + 1} + C$$

$$28) \int \frac{x^5}{(x^3 + 1)(x^3 + 8)} dx = -\frac{1}{21} \text{Log}|x^3 + 1| + \frac{8}{21} \text{Log}|x^3 + 8| + C$$

$$29) \int \frac{x^7 - x^3}{x^{12} - 2x^4 - 1} dx = -\frac{1}{2} \text{Log}(x^4 + 1) + \frac{1}{4} \text{Log}|x^8 - x^4 + 1| + \\ + \frac{1}{2\sqrt{5}} \text{Log} \left| \frac{2x^4 - 1 + \sqrt{5}}{2x^4 - 1 - \sqrt{5}} \right| + C$$

$$30) \int \frac{x^2 - x + 14}{(x - 4)^3(x - 2)} dx = 2 \text{Log} \left| \frac{x - 4}{x - 2} \right| + \frac{3}{x - 4} - \frac{13}{2(x - 4)^2} + C$$

$$31) \int \frac{dx}{x(x^7 + 1)} = \text{Log}|x| - \frac{1}{7} \text{Log}|x^7 + 1| + C$$