

Ejercicios propuestos

$$1) \int \frac{dx}{\operatorname{sen} x} = \operatorname{Log} \left| \operatorname{tg} \left(\frac{x}{2} \right) \right| + C$$

$$2) \int \frac{dx}{\operatorname{cos} x} = \operatorname{Log} \left| \frac{1 + \operatorname{tg}(x/2)}{1 - \operatorname{tg}(x/2)} \right| + C$$

$$3) \int \frac{dx}{\operatorname{sen} x + \operatorname{cos} x + 2} = \sqrt{2} \operatorname{arctg} \left(\frac{1 + \operatorname{tg}(x/2)}{\sqrt{2}} \right) + C$$

$$4) \int \frac{dx}{5 + 3 \operatorname{cos} x} = \frac{1}{2} \operatorname{arctg} \left(\frac{\operatorname{tg}(x/2)}{2} \right) + C$$

$$5) \int \frac{\operatorname{sen} x}{1 + \operatorname{sen} x} dx = x + \frac{2}{1 + \operatorname{tg}(x/2)} + C$$

$$6) \int \frac{\operatorname{tg} x}{1 + \operatorname{cos} x} dx = \operatorname{Log} \left| \frac{1 + \operatorname{cos} x}{\operatorname{cos} x} \right| + C$$

$$7) \int \frac{dx}{\operatorname{sen} x \operatorname{cos}^2 x} = \frac{1}{2} \operatorname{Log} \left| \frac{\operatorname{cos} x - 1}{\operatorname{cos} x + 1} \right| + \frac{1}{\operatorname{cos} x} + C$$

$$8) \int \frac{\operatorname{cos} x}{\operatorname{cos}^2 x - 4 \operatorname{sen} x - 6} dx = -\operatorname{arctg}(2 + \operatorname{sen} x) + C$$

Integración de funciones trigonométricas

$$9) \int \frac{\operatorname{sen}^2 x}{\cos^6 x} dx = \frac{1}{3} \operatorname{tg}^3 x + \frac{1}{5} \operatorname{tg}^5 x + C$$

$$10) \int \frac{dx}{\operatorname{sen}^4 x} = -\frac{1}{3} \frac{1}{\operatorname{tg}^3 x} - \frac{1}{\operatorname{tg} x} + C$$

$$11) \int \cos^3 x dx = \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + C$$

$$12) \int \operatorname{tg}^4 x dx = \frac{\operatorname{tg}^3 x}{3} - \operatorname{tg} x + x + C$$

$$13) \int \operatorname{sen}^3 x \cos^2 x dx = -\frac{\cos^3 x}{3} + \frac{1}{5} \cos^5 x + C$$

$$14) \int \operatorname{tg}^5 x dx = \frac{1}{4} \operatorname{tg}^4 x - \frac{1}{2} \operatorname{tg}^2 x - \operatorname{Log}|\cos x| + C$$

$$15) \int \frac{dx}{(\operatorname{sen} x + \cos x)^2} = -\frac{1}{1 + \operatorname{tg} x} + C$$

$$16) \int \frac{dx}{(1 + \operatorname{tg} x)\operatorname{sen}^2 x} = -\operatorname{Log}|\operatorname{tg} x| - \frac{1}{\operatorname{tg} x} + \operatorname{Log}|1 + \operatorname{tg} x| + C$$

$$17) \int \frac{\cos^3 x}{1 - 2\operatorname{sen} x} dx = \frac{1}{4} \operatorname{sen}^2 x + \frac{1}{4} \operatorname{sen} x - \frac{3}{8} \operatorname{Log}|2\operatorname{sen} x - 1| + C$$

$$18) \int \frac{\cos x}{\operatorname{sen}^2 x - 6\operatorname{sen} x + 5} dx = \frac{1}{4} \operatorname{Log} \left| \frac{\operatorname{sen} x - 5}{\operatorname{sen} x - 1} \right| + C$$

$$19) \int \frac{1 - \operatorname{sen} x + \cos x}{1 + \operatorname{sen} x - \cos x} dx = 2 \operatorname{Log} \left| \frac{\operatorname{tg}(x/2)}{1 + \operatorname{tg}(x/2)} \right| - x + C$$

$$20) \int \frac{\cos x}{2\operatorname{sen} x - \cos x + 3} dx =$$

$$= -\frac{x}{5} + \frac{2}{5} \operatorname{Log} \left| \frac{2\operatorname{tg}^2(x/2) + 2\operatorname{tg}(x/2) + 1}{1 + \operatorname{tg}^2(x/2)} \right| + \frac{3}{5} \operatorname{arctg}(2\operatorname{tg}(x/2) + 1) + C$$

$$21) \int \frac{\operatorname{sen} x}{\operatorname{sen} x - 2\cos x + 1} dx = -\frac{2x}{5} - \frac{1}{2} \operatorname{Log}|1 + \operatorname{tg}(x/2)| +$$

$$+ \frac{9}{10} \operatorname{Log}|\operatorname{tg}(x/2) - 1/3| - \frac{1}{5} \operatorname{Log}|1 + \operatorname{tg}^2(x/2)| + C$$

$$22) \int \frac{dx}{2\operatorname{sen} x - 3\cos x + 4} = \operatorname{Log} \left| \frac{\operatorname{tg}(x/2) + 1/3}{\operatorname{tg}(x/2) + 1} \right| + C$$

$$23) \int \frac{\cos^2 x}{2 + \operatorname{sen}^2 x} dx = -x + \sqrt{\frac{3}{2}} \operatorname{arctg} \left(\sqrt{\frac{3}{2}} \operatorname{tg} x \right) + C$$

$$24) \int \frac{2\operatorname{sen}^2 x}{3 - 4\cos^2 x} dx = \frac{x}{2} + \frac{1}{4\sqrt{3}} \operatorname{Log} \left| \frac{\sqrt{3}\operatorname{tg} x - 1}{\sqrt{3}\operatorname{tg} x + 1} \right| + C$$

$$25) \int \frac{\operatorname{sen} x \cos x}{1 + 2\cos^2 x \operatorname{sen}^2 x} dx = \frac{1}{4\sqrt{3}} \operatorname{Log} \left| \frac{2\operatorname{sen}^2 x - 1 + \sqrt{3}}{2\operatorname{sen}^2 x - 1 - \sqrt{3}} \right| + C$$

$$26) \int \frac{\cos x \operatorname{sen}^2 x}{1 + \operatorname{sen}^3 x + 2\cos^2 x \operatorname{sen} x} dx =$$

$$= -\operatorname{Log}|1 + \operatorname{sen} x| + \frac{1}{\sqrt{5}} \operatorname{Log} \left| \frac{2\operatorname{sen} x - 1 + \sqrt{5}}{2\operatorname{sen} x - 1 - \sqrt{5}} \right| + C$$

$$27) \int \frac{dx}{\cos x \sqrt{\cos 2x}} = \operatorname{arcsen}(\operatorname{tg} x) + C$$

Integración de funciones trigonométricas

$$28) \int \frac{dx}{\operatorname{sen}^4 x + \cos^4 x} = \frac{1}{\sqrt{2}} \operatorname{arctg} \left(\frac{\operatorname{tg} 2x}{\sqrt{2}} \right) + C$$

$$29) \int \operatorname{sen}^3 x \sqrt{\cos x} \, dx = -\frac{2}{3} \sqrt{\cos^3 x} + \frac{2}{7} \sqrt{\cos^7 x} + C$$

$$30) \int \frac{dx}{1 + 2\operatorname{sen}^2 x} = \frac{1}{\sqrt{3}} \operatorname{arctg}(\sqrt{3} \operatorname{tg} x) + C$$

$$31) \int \frac{\cos x}{\operatorname{sen}^2 x - 6\operatorname{sen} x + 12} \, dx = \frac{1}{\sqrt{3}} \operatorname{arctg} \left(\frac{\operatorname{sen} x - 3}{\sqrt{3}} \right) + C$$

$$32) \int \frac{\operatorname{sen} x}{\sqrt{\cos^2 x + 4\cos x + 1}} \, dx = -\operatorname{Log} \left| \cos x + 2 + \sqrt{\cos^2 x + 4\cos x + 1} \right| + C$$

$$33) \int \frac{1 + \cos x}{\operatorname{sen} x (\cos^2 x + 2\cos x + 2)} \, dx =$$

$$= \frac{1}{5} \operatorname{Log} \left| \frac{1 - \cos x}{\sqrt{\cos^2 x + 2\cos x + 2}} \right| - \frac{2}{5} \operatorname{arctg}(1 + \cos x) + C$$

$$34) \int \frac{\cos^3 x}{\operatorname{sen}^3 x + \operatorname{sen}^2 x + \operatorname{sen} x} \, dx = \operatorname{Log} \left| \frac{\operatorname{sen} x}{\operatorname{sen}^2 x + \operatorname{sen} x + 1} \right| + C$$

$$35) \int \frac{\operatorname{sen}(x/2) \cos(5x/2)}{\operatorname{sen} 3x} \, dx = \frac{1}{2} x - \frac{1}{4\sqrt{3}} \operatorname{Log} \left| \frac{\sqrt{3} + 2\operatorname{sen} x}{\sqrt{3} - 2\operatorname{sen} x} \right| + C$$