

## Bloque 3. Geometría y Trigonometría

### Tema 5 Trigonometría básica

### Ejercicios resueltos

3.5-1 Determina los valores que puede tomar el seno de  $\alpha$  si  $\cos \alpha = -0.6$ . ¿A qué cuadrante puede pertenecer  $\alpha$ ?

*Solución*

$$\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \operatorname{sen} \alpha = \pm \sqrt{1 - \cos^2 \alpha}$$

$$\operatorname{sen} \alpha = \pm \sqrt{1 - (-0.6)^2} = \pm \sqrt{1 - 0.36} = \pm \sqrt{0.64} = \pm 0.8$$

$$\cos \alpha = -0.6 < 0 \Rightarrow \begin{cases} \alpha \in \text{II} \\ \alpha \in \text{III} \end{cases} \Rightarrow \begin{cases} \operatorname{sen} \alpha = 0.8 > 0 \\ \operatorname{sen} \alpha = -0.8 < 0 \end{cases}$$

3.5-2 Calcula las razones trigonométricas de un ángulo  $\alpha$  sabiendo que su secante vale -45 y que pertenece al segundo cuadrante.

*Solución*

$$\sec \alpha = -45 = \frac{1}{\cos \alpha} \Rightarrow \cos \alpha = \frac{1}{\sec \alpha} = \frac{1}{-45} = -0.022$$

$$\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \operatorname{sen} \alpha = \pm \sqrt{1 - \cos^2 \alpha}$$

$$\operatorname{sen} \alpha = \pm \sqrt{1 - (-0.022)^2} = \pm \sqrt{1 - 0.00049} = \pm \sqrt{0.9995} = \pm 0.99975$$

$$\alpha \in \text{II} \Rightarrow \operatorname{sen} \alpha = 0.99975 \Rightarrow \operatorname{cosec} \alpha = \frac{1}{\operatorname{sen} \alpha} = \frac{1}{0.99975} = 1.00025$$

$$\operatorname{tag} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} = \frac{0.99975}{-0.022} = -45.44 \Rightarrow \operatorname{cotag} \alpha = \frac{1}{\operatorname{tag} \alpha} = \frac{1}{-45.44} = -0.022$$

3.5-3 Obtén la relación que existe entre las razones trigonométricas de un ángulo  $\alpha$  y las del ángulo  $\pi - \alpha$ .

**Solución**

$$\text{sen}(\pi - \alpha) = \text{sen}(\pi) \cdot \cos(\alpha) - \cos(\pi) \cdot \text{sen}(\alpha)$$

$$\left. \begin{array}{l} \text{sen}(\pi) = 0 \\ \cos(\pi) = -1 \end{array} \right\} \Rightarrow \text{sen}(\pi - \alpha) = \text{sen}(\alpha)$$

$$\cos(\pi - \alpha) = \cos(\pi) \cdot \cos(\alpha) + \text{sen}(\pi) \cdot \text{sen}(\alpha)$$

$$\left. \begin{array}{l} \text{sen}(\pi) = 0 \\ \cos(\pi) = -1 \end{array} \right\} \Rightarrow \cos(\pi - \alpha) = -\cos(\alpha)$$

$$\text{tag}(\pi - \alpha) = \frac{\text{tag}(\pi) - \text{tag}(\alpha)}{1 + \text{tag}(\pi) \cdot \text{tag}(\alpha)}$$

$$\text{tag}(\pi) = \frac{\text{sen}(\pi)}{\cos(\pi)} = \frac{0}{-1} = 0 \Rightarrow \text{tag}(\pi - \alpha) = -\text{tag}(\alpha)$$

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3.5-4 Calcula las razones trigonométricas del ángulo  $135^\circ$ .

**Solución**

$$\text{sen}(135^\circ) = \text{sen}(180^\circ - 45^\circ) = \text{sen}(\pi - \pi/4) = \text{sen}(\pi/4) = \frac{\sqrt{2}}{2}$$

$$\cos(135^\circ) = \cos(180^\circ - 45^\circ) = \cos(\pi - \pi/4) = -\cos(\pi/4) = -\frac{\sqrt{2}}{2}$$

$$\text{tag}(135^\circ) = \text{tag}(180^\circ - 45^\circ) = \text{tag}(\pi - \pi/4) = -\text{tag}(\pi/4) = -1$$

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3.5-5 Calcula las razones trigonométricas del ángulo  $120^\circ$ .

**Solución**

$$\operatorname{sen}(120^\circ) = \operatorname{sen}(180^\circ - 60^\circ) = \operatorname{sen}(\pi - \pi/3) = \operatorname{sen}(\pi/3) = \frac{\sqrt{3}}{2}$$

$$\operatorname{cos}(120^\circ) = \operatorname{cos}(180^\circ - 60^\circ) = \operatorname{cos}(\pi - \pi/3) = -\operatorname{cos}(\pi/3) = -\frac{1}{2}$$

$$\operatorname{tag}(120^\circ) = \operatorname{tag}(180^\circ - 60^\circ) = \operatorname{tag}(\pi - \pi/3) = -\operatorname{tag}(\pi/3) = -\sqrt{3}$$

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3.5-6 Calcula las razones trigonométricas del ángulo  $150^\circ$ .

**Solución**

$$\operatorname{sen}(150^\circ) = \operatorname{sen}(180^\circ - 30^\circ) = \operatorname{sen}(\pi - \pi/6) = \operatorname{sen}(\pi/6) = \frac{1}{2}$$

$$\operatorname{cos}(150^\circ) = \operatorname{cos}(180^\circ - 30^\circ) = \operatorname{cos}(\pi - \pi/6) = -\operatorname{cos}(\pi/6) = -\frac{\sqrt{3}}{2}$$

$$\operatorname{tag}(150^\circ) = \operatorname{tag}(180^\circ - 30^\circ) = \operatorname{tag}(\pi - \pi/6) = -\operatorname{tag}(\pi/6) = -\frac{1}{\sqrt{3}}$$

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3.5-7 Sabiendo que  $\operatorname{sen}(\alpha) = \frac{4}{5}$  y que  $\alpha$  pertenece al segundo cuadrante, calcula  $\operatorname{sen}(2\alpha)$ ,  $\operatorname{cos}(2\alpha)$  y  $\operatorname{tag}(2\alpha)$ .

**Solución**

$$\operatorname{sen}^2 \alpha + \operatorname{cos}^2 \alpha = 1 \Rightarrow \operatorname{cos} \alpha = \pm \sqrt{1 - \operatorname{sen}^2 \alpha} = \pm \sqrt{1 - \frac{16}{25}} = \pm \sqrt{\frac{9}{25}} = \pm \frac{3}{5}$$

$$\alpha \in \text{II} \Rightarrow \operatorname{cos} \alpha = -\frac{3}{5} \Rightarrow \operatorname{tag} \alpha = \frac{4/5}{-3/5} = -\frac{4}{3}$$

$$\operatorname{sen}(2\alpha) = 2 \cdot \operatorname{sen} \alpha \cdot \operatorname{cos} \alpha = 2 \cdot \frac{4}{5} \cdot \left(-\frac{3}{5}\right) = -\frac{24}{25}$$

$$\operatorname{cos}(2\alpha) = \operatorname{cos}^2 \alpha - \operatorname{sen}^2 \alpha = \left(-\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$$

$$\operatorname{tag}(2\alpha) = \frac{2 \cdot \operatorname{tag} \alpha}{1 - \operatorname{tag}^2 \alpha} = \frac{2 \cdot (-4/3)}{1 - (-4/3)^2} = \frac{-8/3}{1 - 16/9} = \frac{-8/3}{-7/9} = \frac{-8/3}{-7/9} = \frac{24}{7}$$

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3.5-8 Utilizando las fórmulas del ángulo mitad, encuentra las razones trigonométricas del ángulo  $15^\circ = \pi/12$ .

**Solución**

$$\cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2} \Rightarrow \cos^2(15^\circ) = \frac{1 + \cos(30^\circ)}{2} = \frac{1 + \frac{\sqrt{3}}{2}}{2} = \frac{2 + \sqrt{3}}{4} = 0.933$$

$$\cos(15^\circ) = \pm\sqrt{0.933} = \pm 0.9659 \Rightarrow 15^\circ \in I \Rightarrow \cos(15^\circ) = 0.9659$$

$$\operatorname{sen}^2(\alpha) = \frac{1 - \cos(2\alpha)}{2} \Rightarrow \operatorname{sen}^2(15^\circ) = \frac{1 - \cos(30^\circ)}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{2 - \sqrt{3}}{4} = 0.067$$

$$\operatorname{sen}(15^\circ) = \pm\sqrt{0.067} = \pm 0.2588 \Rightarrow 15^\circ \in I \Rightarrow \operatorname{sen}(15^\circ) = 0.2588$$

$$\operatorname{tag}^2(\alpha) = \frac{1 - \cos(2\alpha)}{1 + \cos(2\alpha)} \Rightarrow \operatorname{tag}^2(15^\circ) = \frac{1 - \cos(30^\circ)}{1 + \cos(30^\circ)} = \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}} = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} = 0.07178$$

$$\operatorname{tag}(15^\circ) = \pm\sqrt{0.07178} = \pm 0.2679 \Rightarrow 15^\circ \in I \Rightarrow \operatorname{tag}(15^\circ) = 0.2679$$

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