

Bloque I. Función real de variable real  
**Tema 3 Cálculo diferencial**

## Ejercicios resueltos

I.3-1 Deriva las siguientes expresiones:

a)  $y = \operatorname{sen}2x$ ;    b)  $y = \cos(x-1)$ ;    c)  $y = \cos3x^2$ ;

d)  $y = \frac{x}{\sqrt[3]{x^2+4}}$ ;    e)  $y = \left(\frac{3x-1}{x^2+3}\right)^2$ ;    f)  $y = \sqrt{\operatorname{sen}4x}$

### Solución

a)  $y = \operatorname{sen}2x$ . Derivada del seno por la derivada del ángulo:

$$y = \operatorname{sen}2x \Rightarrow y' = \cos 2x \cdot (2x)' \Rightarrow y' = 2 \cdot \cos 2x$$

b)  $y = \cos(x-1)$ . Derivada del coseno por la derivada del ángulo:

$$y = \cos(x-1) \Rightarrow y' = -\operatorname{sen}(x-1) \cdot (x-1)' \Rightarrow y' = -\operatorname{sen}(x-1)$$

c)  $y = \cos3x^2$ . Derivada del coseno por la derivada del ángulo:

$$y = \cos3x^2 \Rightarrow y' = -\operatorname{sen}3x^2 \cdot (3x^2)' \Rightarrow y' = -6x \cdot \operatorname{sen}3x^2$$

d)  $y = \frac{x}{\sqrt[3]{x^2+4}}$ . Derivada de un cociente:

$$y = \frac{x}{\sqrt[3]{x^2+4}} = \frac{x}{(x^2+4)^{1/3}}$$

$$y' = \frac{(x^2+4)^{1/3} - \left((x^2+4)^{1/3}\right)' \cdot x}{\left((x^2+4)^{1/3}\right)^2} = \frac{(x^2+4)^{1/3} - \frac{1}{3}x \cdot (x^2+4)^{-2/3} \cdot 2x}{(x^2+4)^{2/3}} =$$

$$= \frac{1}{3}(x^2+4)^{-2/3} \left[ \frac{3(x^2+4) - 2x^2}{(x^2+4)^{2/3}} \right] = \frac{x^2+12}{3(x^2+4)^{4/3}}$$

e)  $y = \left(\frac{3x-1}{x^2+3}\right)^2$ . Derivada de una potencia:

$$\begin{aligned}y' &= 2 \cdot \frac{3x-1}{x^2+3} \cdot \left(\frac{3x-1}{x^2+3}\right)' = \\ &= 2 \cdot \frac{3x-1}{x^2+3} \cdot \frac{3 \cdot (x^2+3) - 2x \cdot (3x-1)}{(x^2+3)^2} = \frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}\end{aligned}$$

f)  $y = \sqrt{\operatorname{sen}4x}$ . Derivada de una raíz cuadrada:

$$\begin{aligned}y &= \sqrt{\operatorname{sen}4x} \Rightarrow y = (\operatorname{sen}4x)^{\frac{1}{2}} \\ y' &= \frac{1}{2} \cdot (\operatorname{sen}4x)^{-\frac{1}{2}} \cdot (\operatorname{sen}4x)' = \frac{1}{2} (\operatorname{sen}4x)^{-\frac{1}{2}} \cdot 4 \cos 4x = \frac{2 \cos 4x}{\sqrt{\operatorname{sen}4x}}\end{aligned}$$

I.3-2 Derivar:

$$y = x^x$$

### Solución

Aplicamos logaritmos a los dos miembros de la igualdad:

$$\ln y = \ln x^x \Rightarrow \ln y = x \ln x$$

Derivamos los dos miembros de esta última igualdad:

$$(\ln y)' = (x \ln x)' \Rightarrow \frac{y'}{y} = \ln x + \frac{x}{x} \Rightarrow y' = y \cdot (\ln x + 1)$$

$$y' = x^x \cdot (\ln x + 1)$$

I.3-3 Derivar:

$$y = \frac{2}{\sqrt{3}} \operatorname{arctag} \left( \frac{\operatorname{tag} \left( \frac{x}{2} \right)}{\sqrt{3}} \right) - \frac{1}{\sqrt{2}} \operatorname{arctag} \left( \frac{\operatorname{tag} \left( \frac{x}{2} \right)}{\sqrt{2}} \right)$$

### Solución

$$\begin{aligned}y' &= \frac{2}{\sqrt{3}} \frac{1}{1 + \frac{1}{3} \operatorname{tg}^2\left(\frac{x}{2}\right)} \cdot \frac{1}{2\sqrt{3} \cos^2\left(\frac{x}{2}\right)} - \frac{1}{\sqrt{2}} \frac{1}{1 + \frac{1}{2} \operatorname{tg}^2\left(\frac{x}{2}\right)} \cdot \frac{1}{2\sqrt{2} \cos^2\left(\frac{x}{2}\right)} = \\&= \frac{1}{\left(3 + \operatorname{tg}^2\left(\frac{x}{2}\right)\right) \cos^2\left(\frac{x}{2}\right)} - \frac{1}{\left(4 + 2\operatorname{tg}^2\left(\frac{x}{2}\right)\right) \cos^2\left(\frac{x}{2}\right)} = \\&= \frac{1}{3 \cos^2\left(\frac{x}{2}\right) + \operatorname{tg}^2\left(\frac{x}{2}\right) \cdot \cos^2\left(\frac{x}{2}\right)} - \frac{1}{4 \cos^2\left(\frac{x}{2}\right) + 2\operatorname{tg}^2\left(\frac{x}{2}\right) \cdot \cos^2\left(\frac{x}{2}\right)}\end{aligned}$$

Hemos de tener en cuenta que:

$$\operatorname{tg}^2\left(\frac{x}{2}\right) = \frac{\operatorname{sen}^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} \Rightarrow \operatorname{tg}^2\left(\frac{x}{2}\right) \cdot \cos^2\left(\frac{x}{2}\right) = \frac{\operatorname{sen}^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} \cdot \cos^2\left(\frac{x}{2}\right) = \operatorname{sen}^2\left(\frac{x}{2}\right)$$

Por lo tanto:

$$\begin{aligned}y' &= \frac{1}{3 \cos^2\left(\frac{x}{2}\right) + \operatorname{sen}^2\left(\frac{x}{2}\right)} - \frac{1}{4 \cos^2\left(\frac{x}{2}\right) + 2\operatorname{sen}^2\left(\frac{x}{2}\right)} = \\&= \left\{ \cos^2\left(\frac{x}{2}\right) + \operatorname{sen}^2\left(\frac{x}{2}\right) = 1 \right\} = \\&= \frac{1}{1 + 2 \cos^2\left(\frac{x}{2}\right)} - \frac{1}{2 + 2 \cos^2\left(\frac{x}{2}\right)} = \left\{ \cos^2\left(\frac{x}{2}\right) = \frac{1 + \cos x}{2} \right\} = \\&= \frac{1}{1 + 1 + \cos x} - \frac{1}{2 + 1 + \cos x} = \frac{3 + \cos x - (2 + \cos x)}{(2 + \cos x)(3 + \cos x)} = \frac{1}{(2 + \cos x)(3 + \cos x)}\end{aligned}$$

I.3-4 Derivar:

$$y = \operatorname{arc.} \cos\left(\frac{1-x^2}{1+x^2}\right)$$

### Solución

Derivada del arco coseno:

$$\begin{aligned} y' &= \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2} = \frac{2x+2x^3+2x-2x^3}{(1+x^2)^2 \sqrt{1-\frac{1-2x^2+x^4}{1+2x^2+x^4}}} = \\ &= \frac{4x}{(1+x^2)^2 \sqrt{\frac{4x^2}{(1+x^2)^2}}} = \frac{4x}{2x(1+x^2)} = \frac{2}{1+x^2} \end{aligned}$$

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I.3-5 Derivar:

$$y = (1+\sqrt{1+x})^{3/2} - 3(1+\sqrt{1+x})^{1/2}$$

### Solución

Derivada de potencias:

$$\begin{aligned} y' &= \frac{3}{2}(1+\sqrt{1+x})^{1/2} \cdot \frac{1}{2\sqrt{1+x}} - \frac{3}{2}(1+\sqrt{1+x})^{-1/2} \cdot \frac{1}{2\sqrt{1+x}} = \\ &= \frac{3}{4\sqrt{1+x}} \left( \sqrt{1+\sqrt{1+x}} - \frac{1}{\sqrt{1+\sqrt{1+x}}} \right) = \frac{3}{4\sqrt{1+x}} \left( \frac{1+\sqrt{1+x}-1}{\sqrt{1+\sqrt{1+x}}} \right) = \\ &= \frac{3}{4\sqrt{1+\sqrt{1+x}}} \end{aligned}$$

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I.3-6 Derivar:

$$y = \ln \left[ \frac{1 - \operatorname{tag}^2\left(\frac{x}{2}\right)}{1 + \operatorname{tag}^2\left(\frac{x}{2}\right)} \cdot e^{x \cos x} \right]$$

### Solución

Antes de derivar operamos:

$$\frac{1 - \operatorname{tag}^2\left(\frac{x}{2}\right)}{1 + \operatorname{tag}^2\left(\frac{x}{2}\right)} = \frac{1 - \frac{\operatorname{sen}^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}}{1 + \frac{\operatorname{sen}^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}} = \frac{\frac{\cos^2\left(\frac{x}{2}\right) - \operatorname{sen}^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}}{\frac{\cos^2\left(\frac{x}{2}\right) + \operatorname{sen}^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}} =$$

$$= \frac{\cos^2\left(\frac{x}{2}\right) - \operatorname{sen}^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right) + \operatorname{sen}^2\left(\frac{x}{2}\right)} = \cos x$$

Sustituyendo este valor, nuestro problema se convierte en:

$$y = \ln[\cos x \cdot e^{x \cos x}] = \ln(\cos x) + \ln(e^{x \cos x}) = x \cos x + \ln(\cos x)$$

Ahora derivamos esta expresión:

$$y' = \cos x - x \operatorname{sen} x + \frac{-\operatorname{sen} x}{\cos x} \Rightarrow y' = \cos x - x \operatorname{sen} x - \operatorname{tag} x$$

I.3-7 Derivar:

$$y = \operatorname{arctag} x + \operatorname{arctag}\left(\frac{1}{x}\right) + \operatorname{arcsen} x + \operatorname{arccos} \sqrt{1-x^2}$$

**Solución**

$$y' = \frac{1}{1+x^2} + \frac{1}{1+\left(\frac{1}{x^2}\right)} \cdot \frac{-1}{x^2} + \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-(1-x^2)}} \cdot \frac{-2x}{2\sqrt{1-x^2}} =$$

$$= \frac{1}{1+x^2} - \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x^2}} \cdot \frac{x}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} =$$

$$= \frac{2}{\sqrt{1-x^2}}$$

I.3-8 Derivar:

$$y = \ln \sqrt[3]{1-x^4}$$

**Solución**

$$y = \ln \sqrt[3]{1-x^4} \Rightarrow y = \frac{1}{3} \ln(1-x^4)$$

Derivamos esta última expresión:

$$y' = \frac{1}{3} \cdot \frac{1}{(1-x^4)} \cdot (-4x^3) = -\frac{4x^3}{3(1-x^4)}$$

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