Ejercicios resueltos

II.2-1 Aproxima el valor de las siguientes integrales definidas por los métodos del rectángulo, del punto medio, del trapecio y de Simpson, tomando para todos los casos el mismo valor de h = 0.1. Calcula el error que se comete en cada caso en relación con el valor exacto que se proporciona.

$a)\ \int_0^2 3x^2 dx = 8$

$b)\ \int_0^1 e^x dx = 1.71828$

$c)\ \int_{-1}^1 \left(x + 2x^2 - x^3 + 5x^4\right) dx = 3.3333$

$d)\ \int_2^3 \frac{1}{\sqrt{x-1}} dx = 0.828427$

$e)\ \int_1^2 \frac{2x+1}{x^2+x} dx = 1.09861$

$f)\ \int_0^1 \frac{1}{1+x^2} dx = 0.785398$

Solución

$a)\ \int_0^2 3x^2 dx = 8 \qquad b-a = nh \Rightarrow 2 = 0.1n \Rightarrow n = 20$

Rectángulo: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f(x_i)$

$$\int_0^2 3x^2 dx \approx 0.1\left[3(0)^2 + 3(0.1)^2 + \ldots + 3(1.8)^2 + 3(1.9)^2\right] = 7.41$$

$$ERROR = |8 - 7.41| = 0.59$$

Punto medio: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f\left(x_0 + \frac{2i+1}{2}h\right)$

$$\int_0^2 3x^2 dx \approx 0.1\left[3(0.05)^2 + 3(0.15)^2 + \ldots + 3(1.85)^2 + 3(1.95)^2\right] = 7.995$$

$$ERROR = |8 - 7.995| = 0.005$$

Trapecio: $\int_a^b f(x)\,dx \approx \frac{h}{2}\big(f(a) + f(b)\big) + h\sum_{i=1}^{n-1} f(x_i)$

$$\int_0^2 3x^2dx \approx \frac{0.1}{2}\left[3(0)^2+3(2)^2\right]+0.1\left[3(0.1)^2+3(0.2)^2+\ldots+3(1.8)^2+3(1.9)^2\right]=8.01$$

$$ERROR=|8-8.01|=0.01$$

Simpson: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a)+f(b)\right)+\frac{2h}{3}\sum_{i=1}^{n-1} f\left(x_{2i}\right)+\frac{4h}{3}\sum_{i=1}^{n} f\left(x_{2i-1}\right)$

$$\int_0^2 3x^2dx \approx \frac{0.1}{3}\left[3(0)^2+3(2)^2\right]+\frac{0.2}{3}\left[3(0.2)^2+3(0.4)^2+\ldots+3(1.6)^2+3(1.8)^2\right]+$$
$$+\frac{0.4}{3}\left[3(0.1)^2+3(0.3)^2+\ldots+3(1.7)^2+3(1.9)^2\right]=8$$

$$ERROR=|8-8|=0$$

$b)$ $\int_0^1 e^x dx = 1.71828$ $\qquad$ $b-a=nh \Rightarrow 1=0.1n \Rightarrow n=10$

Rectángulo: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f\left(x_i\right)$

$$\int_0^1 e^x dx \approx 0.1\left[e^0+e^{0.1}+\ldots+e^{0.8}+e^{0.9}\right]=1.6337994$$

$$ERROR=|1.71828-1.6337994|=0.0844806$$

Punto medio: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f\left(x_0+\frac{2i+1}{2}h\right)$

$$\int_0^1 e^x dx \approx 0.1\left[e^{0.05}+e^{0.15}+\ldots+e^{0.85}+e^{0.95}\right]=1.717566086$$

$$ERROR=|1.71828-1.717566086|=0.000713914$$

Trapecio: $\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a)+f(b)\right)+h\sum_{i=1}^{n-1} f\left(x_i\right)$

$$\int_0^1 e^x dx \approx \frac{0.1}{2}\left[e^0+e^1\right]+0.1\left[e^{0.1}+e^{0.2}+\ldots+e^{0.8}+e^{0.9}\right]=1.6337994$$

$$ERROR=|1.71828-1.6337994|=0.0844806$$

Simpson: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a)+f(b)\right)+\frac{2h}{3}\sum_{i=1}^{n-1} f\left(x_{2i}\right)+\frac{4h}{3}\sum_{i=1}^{n} f\left(x_{2i-1}\right)$

$$\int_0^1 e^x dx \approx \frac{0.1}{3}\left[e^0 + e^1\right] + \frac{0.2}{3}\left[e^{0.2} + e^{0.4} + e^{0.6} + e^{0.8}\right] +$$

$$+\frac{0.4}{3}\left[e^{0.1} + e^{0.3} + e^{0.5} + e^{0.7} + e^{0.9}\right] = 1.661006721$$

$$ERROR = |1.71828 - 1.661006721| = 0.0572733$$

c) $\int_{-1}^1 \left(x + 2x^2 - x^3 + 5x^4\right)dx = 3.3333$ $\qquad b - a = nh \Rightarrow 2 = 0.1n \Rightarrow n = 20$

Rectángulo: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f(x_i)$

$$\int_{-1}^1 \left(x + 2x^2 - x^3 + 5x^4\right)dx \approx 0.1\left[f(-1) + f(-0.9) + \ldots + f(0.8) + f(0.9)\right] = 3.3733$$

$$ERROR = |3.3733 - 3.3333| = 0.04$$

Punto medio: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f\left(x_0 + \frac{2i+1}{2}h\right)$

$$\int_{-1}^1 \left(x + 2x^2 - x^3 + 5x^4\right)dx \approx 0.1\left[f(-0.95) + f(-0.85) + \ldots + f(0.85) + f(0.95)\right] =$$
$$= 3.3133625$$

$$ERROR = |3.3333333 - 3.3133625| = 0.0199708$$

Trapecio: $\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a) + f(b)\right) + h\sum_{i=1}^{n-1} f(x_i)$

$$\int_{-1}^1 \left(x + 2x^2 - x^3 + 5x^4\right)dx \approx \frac{0.1}{2}\left[f(-1) + f(1)\right] +$$

$$+0.1\left[f(-0.9) + f(-0.8) + \ldots + f(0.8) + f(0.9)\right] = 3.3733$$

$$ERROR = |3.3733 - 3.3333| = 0.04$$

Simpson: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a) + f(b)\right) + \frac{2h}{3}\sum_{i=1}^{n-1} f\left(x_{2i}\right) + \frac{4h}{3}\sum_{i=1}^{n} f\left(x_{2i-1}\right)$

$$\int_{-1}^1 \left(x + 2x^2 - x^3 + 5x^4\right)dx \approx \frac{0.1}{3}\left[f(-1) + f(1)\right] +$$

$$+\frac{0.2}{3}\left[f(-0.8) + f(-0.6) + \ldots + f(0.6) + f(0.8)\right] +$$

$$+\frac{0.4}{3}\left[f(-0.9) + f(-0.7) + \ldots + f(0.7) + f(0.9)\right] = 3.333466667$$

$$ERROR = |3.333466667 - 3.333333333| = 0.000133334$$

d) $\int_2^3 \frac{1}{\sqrt{x-1}}dx = 0.828427$ $\qquad$ $b - a = nh \Rightarrow 1 = 0.1n \Rightarrow n = 10$

Rectángulo: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f(x_i)$

$$\int_2^3 \frac{1}{\sqrt{x-1}}dx \approx 0.1\left[f(2) + f(2.1) + \ldots + f(2.8) + f(2.9)\right] = 0.843340902$$

$$ERROR = |0.843340902 - 0.828427| = 0.014913902$$

Punto medio: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f\left(x_0 + \frac{2i+1}{2}h\right)$

$$\int_2^3 \frac{1}{\sqrt{x-1}}dx \approx 0.1\left[f(2.05) + f(2.15) + \ldots + f(2.85) + f(2.95)\right] =$$
$$= 0.828292655$$

$$ERROR = |0.828292655 - 0.828427| = 0.000134345$$

Trapecio: $\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a) + f(b)\right) + h\sum_{i=1}^{n-1} f(x_i)$

$$\int_2^3 \frac{1}{\sqrt{x-1}}dx \approx \frac{0.1}{2}\left[f(2) + f(3)\right] +$$
$$+0.1\left[f(2.1) + f(2.2) + \ldots + f(2.8) + f(2.9)\right] = 0.828696241$$

$$ERROR = |0.828696241 - 0.88427| = 0.000269241$$

Simpson: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a) + f(b)\right) + \frac{2h}{3}\sum_{i=1}^{n-1} f(x_{2i}) + \frac{4h}{3}\sum_{i=1}^{n} f(x_{2i-1})$

$$\int_2^3 \frac{1}{\sqrt{x-1}}dx \approx \frac{0.1}{3}\left[f(2) + f(3)\right] +$$
$$+\frac{0.2}{3}\left[f(2.2) + f(2.4) + f(2.6) + f(2.8)\right] +$$
$$+\frac{0.4}{3}\left[f(2.1) + f(2.3) + f(2.5) + f(2.7) + f(2.9)\right] = 0.828428056$$

$$ERROR = |0.828428056 - 0.828427| = 0.00000105593$$

$e)\ \int_1^2 \frac{2x+1}{x^2+x}dx = 1.09861$ $\qquad b-a=nh \Rightarrow 1=0.1n \Rightarrow n=10$

Rectángulo: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f(x_i)$

$$\int_1^2 \frac{2x+1}{x^2+x}dx \approx 0.1\left[f(1)+f(1.1)+\ldots+f(1.8)+f(1.9)\right] = 1.132685544$$

$$ERROR = |1.132685544 - 1.09861| = 0.034075544$$

Punto medio: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f\left(x_0+\frac{2i+1}{2}h\right)$

$$\int_1^2 \frac{2x+1}{x^2+x}dx \approx 0.1\left[f(1.05)+f(1.15)+\ldots+f(1.85)+f(1.95)\right] =$$
$$= 1.098242635$$

$$ERROR = |1.098242635 - 1.09861| = 0.000367365$$

Trapecio: $\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a)+f(b)\right)+h\sum_{i=1}^{n-1} f(x_i)$

$$\int_1^2 \frac{2x+1}{x^2+x}dx \approx \frac{0.1}{2}\left[f(1)+f(2)\right]+$$
$$+0.1\left[f(1.1)+f(1.2)+\ldots+f(1.8)+f(1.9)\right] = 1.09935221$$

$$ERROR = |1.09935221 - 1.09861| = 0.00074221$$

Simpson: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a)+f(b)\right)+\frac{2h}{3}\sum_{i=1}^{n-1} f(x_{2i})+\frac{4h}{3}\sum_{i=1}^{n} f(x_{2i-1})$

$$\int_1^2 \frac{2x+1}{x^2+x}dx \approx \frac{0.1}{3}\left[f(1)+f(2)\right]+$$
$$+\frac{0.2}{3}\left[f(1.2)+f(1.4)+f(1.6)+f(1.8)\right]+$$
$$+\frac{0.4}{3}\left[f(1.1)+f(1.3)+f(1.5)+f(1.7)+f(1.9)\right] = 1.098615505$$

$$ERROR = |1.098615505 - 1.09861| = 0.00000550486$$

$f)\ \int_0^1 \frac{1}{1+x^2}\,dx = 0.785398$ $\qquad b-a = nh \Rightarrow 1 = 0.1n \Rightarrow n = 10$

Rectángulo: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f(x_i)$

$$\int_0^1 \frac{1}{1+x^2}\,dx \approx 0.1\left[f(0)+f(0.1)+\ldots+f(0.8)+f(0.9)\right] = 0.809981497$$

$$ERROR = |0.809981497 - 0.785398| = 0.024583497$$

Punto medio: $\int_a^b f(x)\,dx \approx h\sum_{i=0}^{n-1} f\left(x_0 + \frac{2i+1}{2}h\right)$

$$\int_0^1 \frac{1}{1+x^2}\,dx \approx 0.1\left[f(0.05)+f(0.15)+\ldots+f(0.85)+f(0.95)\right] =$$
$$= 0.785606496$$

$$ERROR = |0.785606496 - 0.785398| = 0.000208496$$

Trapecio: $\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a)+f(b)\right) + h\sum_{i=1}^{n-1} f(x_i)$

$$\int_0^1 \frac{1}{1+x^2}\,dx \approx \frac{0.1}{2}\left[f(0)+f(1)\right] +$$
$$+0.1\left[f(0.1)+f(0.2)+\ldots+f(0.8)+f(0.9)\right] = 0.784981497$$

$$ERROR = |0.784981497 - 0.785398| = 0.000416503$$

Simpson: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a)+f(b)\right) + \frac{2h}{3}\sum_{i=1}^{n-1} f(x_{2i}) + \frac{4h}{3}\sum_{i=1}^{n} f(x_{2i-1})$

$$\int_0^1 \frac{1}{1+x^2}\,dx \approx \frac{0.1}{3}\left[f(0)+f(1)\right] +$$
$$+\frac{0.2}{3}\left[f(0.2)+f(0.4)+f(0.6)+f(0.8)\right] +$$
$$+\frac{0.4}{3}\left[f(0.1)+f(0.3)+f(0.5)+f(0.7)+f(0.9)\right] = 0.785398153$$

$$ERROR = |0.785398153 - 0.785398| = 0.000000153485$$

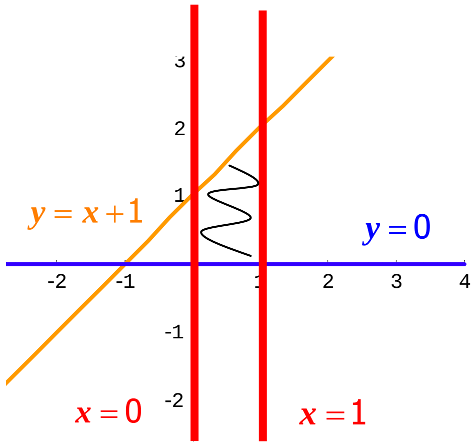
II.2-2 Calcula el área de la región limitada por las siguientes gráficas utilizando el método de Simpson con h = 0.01 para obtener la integral definida correspondiente.

$$a)\ \left.\begin{array}{l} y = x+1 \\ y = 0\ (EJE\ \ OX) \\ x = 0 \\ x = 1 \end{array}\right\} \qquad\qquad b)\ \left.\begin{array}{l} y = x^2+1 \\ y = 0\ (EJE\ \ OX) \\ x = 1 \\ x = 2 \end{array}\right\}$$

Solución

$$a)\ \left.\begin{array}{l} y = x+1 \\ y = 0\ (EJE\ \ OX) \\ x = 0 \\ x = 1 \end{array}\right\} \qquad A = \int_0^1 (x+1)\,dx$$

$$b-a = nh \Rightarrow 1 = 0.01n \Rightarrow n = 100$$

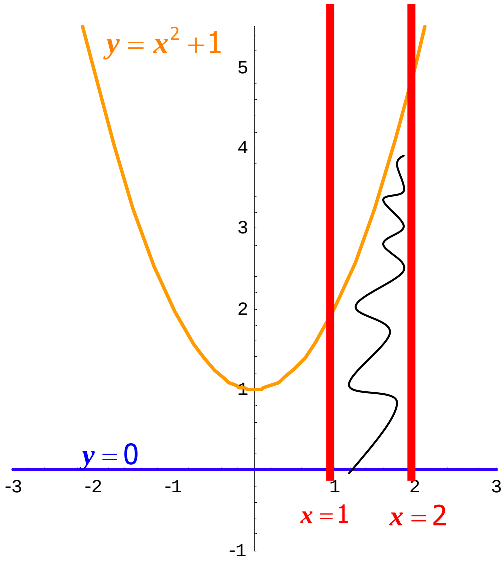


Simpson:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\big(f(a)+f(b)\big)+\frac{2h}{3}\sum_{i=1}^{n-1} f\left(x_{2i}\right)+\frac{4h}{3}\sum_{i=1}^{n} f\left(x_{2i-1}\right)$$

$$\int_0^1 (x+1)\,dx \approx \frac{0.01}{3}\left[f(0)+f(1)\right]+$$
$$+\frac{0.02}{3}\left[f(0.02)+f(0.04)+\ldots+f(0.96)+f(0.98)\right]+$$
$$+\frac{0.04}{3}\left[f(0.01)+f(0.03)+\ldots+f(0.97)+f(0.99)\right]=1.5$$

$$b)\quad \left.\begin{array}{l} y = x^2+1 \\ y = 0 \ (EJE\ \ OX) \\ x = 1 \\ x = 2 \end{array}\right\} \qquad A = \int_1^2 \left(x^2+1\right) dx$$



$$b - a = nh \Rightarrow 1 = 0.01n \Rightarrow n = 100$$

Simpson:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\big(f(a)+f(b)\big) + \frac{2h}{3}\sum_{i=1}^{n-1} f\left(x_{2i}\right) + \frac{4h}{3}\sum_{i=1}^{n} f\left(x_{2i-1}\right)$$

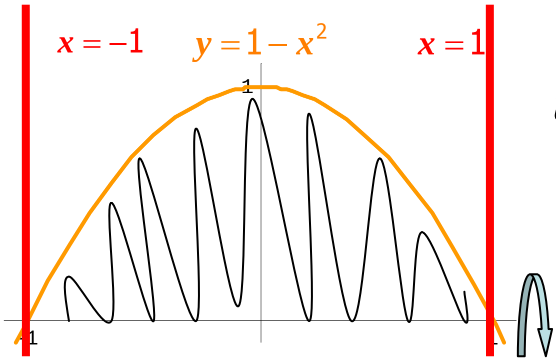
$$\int_1^2 \left(x^2+1\right) dx \approx \frac{0.01}{3}\left[f(1)+f(2)\right]+$$
$$+\frac{0.02}{3}\left[f(1.02)+f(1.04)+\ldots+f(1.96)+f(1.98)\right]+$$
$$+\frac{0.04}{3}\left[f(1.01)+f(1.03)+\ldots+f(1.97)+f(1.99)\right] = 3.333333333$$

II.2-3 Calcula el volumen del sólido de rev olución generado al girar alrededor del eje OX las siguientes gráficas. Utiliza el método de Simpson con h = 0.1 para obtener la integral definida correspondiente.

$$a)\quad \left.\begin{array}{l} y = 1-x^2 \\ x = -1 \\ x = 1 \end{array}\right\} \qquad\qquad b)\quad \left.\begin{array}{l} y = -\frac{1}{2}x^2+2x \\ y = \frac{1}{2}x \end{array}\right\}$$

Solución

$$a)\quad \left.\begin{array}{l} y = 1-x^2 \\ x = -1 \\ x = 1 \end{array}\right\} \qquad V = \pi\int_{-1}^{1}\left(1-x^2\right)^2 dx$$



$$b-a=nh \Rightarrow 2=0.1n \Rightarrow n=20$$

Simpson:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a)+f(b)\right)+\frac{2h}{3}\sum_{i=1}^{n-1} f(x_{2i})+\frac{4h}{3}\sum_{i=1}^{n} f(x_{2i-1})$$

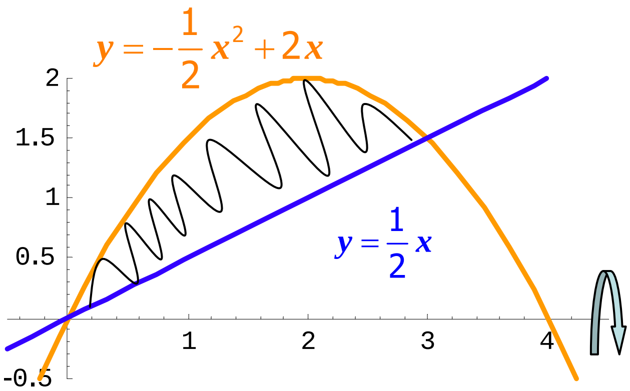
$$\int_{-1}^{1}\left(1-x^2\right)^2 dx \approx \frac{0.1}{3}\left[f(-1)+f(1)\right]+$$

$$+\frac{0.2}{3}\left[f(-0.8)+f(-0.6)+\ldots+f(0.6)+f(0.8)\right]+$$

$$+\frac{0.4}{3}\left[f(-0.9)+f(-0.7)+\ldots+f(0.7)+f(0.9)\right]=1.066693333$$

$$V=1.066693333\pi$$

b) $\left.\begin{array}{l} y=-\dfrac{1}{2}x^2+2x \\ y=\dfrac{1}{2}x \end{array}\right\}$



Puntos de corte:

$$-\frac{x^2}{2}+2x=\frac{x}{2} \Rightarrow -x^2+4x=x \Rightarrow x^2-3x=0 \Rightarrow x\cdot(x-3)=0 \Rightarrow x=0,3$$

Se calcula el volumen generado por la gráfica de arriba (volumen lleno) y se le resta el volume n generado po r la gr áfica de abajo (volumen del agujero):

$$V=\pi\int_0^3\left(-\frac{x^2}{2}+2x\right)^2 dx-\pi\int_0^3\left(\frac{x}{2}\right)^2 dx=\pi\int_0^3\left(\frac{x^4}{4}-2x^3+4x^2\right)dx-\pi\int_0^3\frac{x^2}{4}\,dx=$$

$$=\pi\int_0^3\left(\frac{x^4}{4}-2x^3+\frac{15}{4}x^2\right)dx$$

$b - a = nh \Rightarrow 3 = 0.1n \Rightarrow n = 30$

Simpson:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\big(f(a) + f(b)\big) + \frac{2h}{3}\sum_{i=1}^{n-1} f\left(x_{2i}\right) + \frac{4h}{3}\sum_{i=1}^{n} f\left(x_{2i-1}\right)$$

$$\int_0^3 \left(\frac{x^4}{4} - 2x^3 + \frac{15}{4}x^2\right)dx \approx \frac{0.1}{3}\left[f(3) + f(0)\right] +$$

$$+\frac{0.2}{3}\left[f(0.2) + f(0.4) + \ldots + f(2.6) + f(2.8)\right] +$$

$$+\frac{0.4}{3}\left[f(0.1) + f(0.3) + \ldots + f(2.7) + f(2.9)\right] = 5.40001$$

$V = 5.40001\pi$