

Bloque II. Aproximación Numérica

Tema 4 Interpolación

Ejercicios resueltos

II.4-1 Estimar $f(1)$ a partir de los datos experimentales de la tabla utilizando interpolación lineal, parabólica y de Lagrange.

x	-1	0	3	7
f(x)	2	0	4	7

Solución

LINEAL:

$$p(x) = y_0 \frac{x - x_1}{x_0 - x_1} + y_1 \frac{x - x_0}{x_1 - x_0}$$

Tomamos los puntos $x = 0$ y $x = 3$.

$$\left. \begin{array}{l} x_0 = 0 \Rightarrow y_0 = 0 \\ x_1 = 3 \Rightarrow y_1 = 4 \end{array} \right\}$$

$$p(x) = 0 \frac{x - 3}{0 - 3} + 4 \frac{x - 0}{3 - 0} = \frac{4}{3}x \Rightarrow p(1) = \frac{4}{3} = 1.33333333 \approx f(1)$$

PARABÓLICA:

$$p(x) = y_0 \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + y_1 \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + y_2 \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

Tomamos los puntos $x = 0$, $x = 3$ y $x = 7$.

$$\left. \begin{array}{l} x_0 = 0 \Rightarrow y_0 = 0 \\ x_1 = 3 \Rightarrow y_1 = 4 \\ x_2 = 7 \Rightarrow y_2 = 7 \end{array} \right\}$$

$$p(x) = 0 \frac{(x - 3)(x - 7)}{(0 - 3)(0 - 7)} + 4 \frac{(x - 0)(x - 7)}{(3 - 0)(3 - 7)} + 7 \frac{(x - 0)(x - 3)}{(7 - 0)(7 - 3)}$$

$$p(x) = 4 \frac{x^2 - 7x}{-12} + 7 \frac{x^2 - 3x}{28} = \frac{7x - x^2}{3} + \frac{x^2 - 3x}{4} = \frac{19x - x^2}{12}$$

$$p(1) = \frac{19-1}{12} = \frac{18}{12} = \frac{3}{2} = 1.5 \approx f(1)$$

LAGRANGE:

$$p(x) = \sum_{i=0}^n L_i(x) f(x_i)$$

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j} = \frac{(x - x_0)(x - x_1) \cdots (x - x_n)}{(x_i - x_0)(x_i - x_1) \cdots (x_i - x_n)}$$

$$\left. \begin{array}{l} x_0 = -1 \Rightarrow y_0 = 2 \\ x_1 = 0 \Rightarrow y_1 = 0 \\ x_2 = 3 \Rightarrow y_2 = 4 \\ x_3 = 7 \Rightarrow y_3 = 7 \end{array} \right\}$$

$$p(x) = \sum_{i=0}^3 L_i(x) f(x_i) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2) + L_3(x) f(x_3)$$

$$\left. \begin{array}{l} L_0(x) = \frac{(x-0)(x-3)(x-7)}{(-1-0)(-1-3)(-1-7)} = -\frac{(x-0)(x-3)(x-7)}{32} \\ L_1(x) = \frac{(x+1)(x-3)(x-7)}{(0+1)(0-3)(0-7)} = \frac{(x+1)(x-3)(x-7)}{21} \\ L_2(x) = \frac{(x+1)(x-0)(x-7)}{(3+1)(3-0)(3-7)} = -\frac{(x+1)(x-0)(x-7)}{48} \\ L_3(x) = \frac{(x+1)(x-0)(x-3)}{(7+1)(7-0)(7-3)} = \frac{(x+1)(x-0)(x-3)}{224} \end{array} \right\}$$

$$p(1) = -\frac{3}{8} \cdot 2 + \frac{24}{21} \cdot 0 + \frac{1}{4} \cdot 4 - \frac{1}{56} \cdot 7 = -\frac{3}{4} + 1 - \frac{7}{56} = \frac{-42 + 56 - 7}{56} = \frac{7}{56} = 0.125$$

II.4-2 Estimar $f(0)$ y $f(2)$ a partir de los datos experimentales de la tabla utilizando interpolación lineal, parabólica y de Lagrange.

x	-3	-1	1	3	5	7
f(x)	14	4	2	8	22	44

Solución

LINEAL:

$$p(x) = y_0 \frac{x - x_1}{x_0 - x_1} + y_1 \frac{x - x_0}{x_1 - x_0}$$

Para $x = 0$, tomamos los puntos $x = -1$ y $x = 1$.

$$\left. \begin{array}{l} x_0 = -1 \Rightarrow y_0 = 4 \\ x_1 = 1 \Rightarrow y_1 = 2 \end{array} \right\}$$

$$p(x) = 4 \frac{x - 1}{-1 - 1} + 2 \frac{x + 1}{1 + 1} = -2(x - 1) + x + 1 \Rightarrow p(0) = 2 + 1 = 3 \approx f(0)$$

Para $x = 2$, tomamos los puntos $x = 1$ y $x = 3$.

$$\left. \begin{array}{l} x_0 = 1 \Rightarrow y_0 = 2 \\ x_1 = 3 \Rightarrow y_1 = 8 \end{array} \right\}$$

$$p(x) = 2 \frac{x - 3}{1 - 3} + 8 \frac{x - 1}{3 - 1} = -(x - 3) + 4(x - 1) \Rightarrow p(2) = 1 + 4 = 5 \approx f(2)$$

PARABÓLICA:

$$p(x) = y_0 \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + y_1 \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + y_2 \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

Para $x = 0$, tomamos los puntos $x = -1$, $x = 1$ y $x = 3$

$$\left. \begin{array}{l} x_0 = -1 \Rightarrow y_0 = 4 \\ x_1 = 1 \Rightarrow y_1 = 2 \\ x_2 = 3 \Rightarrow y_2 = 8 \end{array} \right\}$$

$$p(x) = 4 \frac{(x - 1)(x - 3)}{(-1 - 1)(-1 - 3)} + 2 \frac{(x + 1)(x - 3)}{(1 + 1)(1 - 3)} + 8 \frac{(x + 1)(x - 1)}{(3 + 1)(3 - 1)}$$

$$p(x) = \frac{(x-1)(x-3)}{2} - \frac{(x+1)(x-3)}{2} + (x+1)(x-1)$$

$$p(0) = \frac{(0-1)(0-3)}{2} - \frac{(0+1)(0-3)}{2} + (0+1)(0-1) = \frac{3}{2} + \frac{3}{2} - 1 = 2 \approx f(0)$$

Para $x = 2$, tomamos los puntos $x = 1$, $x = 3$ y $x = 5$

$$\left. \begin{array}{l} x_0 = 1 \Rightarrow y_0 = 2 \\ x_1 = 3 \Rightarrow y_1 = 8 \\ x_2 = 5 \Rightarrow y_2 = 22 \end{array} \right\}$$

$$p(x) = 2 \frac{(x-3)(x-5)}{(1-3)(1-5)} + 8 \frac{(x-1)(x-5)}{(3-1)(3-5)} + 22 \frac{(x-1)(x-3)}{(5-1)(5-3)}$$

$$p(x) = \frac{(x-3)(x-5)}{4} - 2(x-1)(x-5) + 11 \frac{(x-1)(x-3)}{4}$$

$$p(2) = \frac{(2-3)(2-5)}{4} - 2(2-1)(2-5) + 11 \frac{(2-1)(2-3)}{4} = \frac{3}{4} + 6 - \frac{11}{4} = 4 \approx f(2)$$

LAGRANGE:

$$p(x) = \sum_{i=0}^n L_i(x) f(x_i), \quad L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j} = \frac{(x - x_0)(x - x_1) \cdots (x - x_n)}{(x_i - x_0)(x_i - x_1) \cdots (x_i - x_n)}$$

$$\left. \begin{array}{l} x_0 = -3 \Rightarrow y_0 = 14 \\ x_1 = -1 \Rightarrow y_1 = 4 \\ x_2 = 1 \Rightarrow y_2 = 2 \\ x_3 = 3 \Rightarrow y_3 = 8 \\ x_4 = 5 \Rightarrow y_4 = 22 \\ x_5 = 7 \Rightarrow y_5 = 44 \end{array} \right\}$$

$$\left. \begin{array}{l} L_0(0) = \frac{(0+1)(0-1)(0-3)(0-5)(0-7)}{(-3+1)(-3-1)(-3-3)(-3-5)(-3-7)} = -\frac{105}{3840} = -\frac{7}{256} \\ L_1(0) = \frac{(0+3)(0-1)(0-3)(0-5)(0-7)}{(-1+3)(-1-1)(-1-3)(-1-5)(-1-7)} = \frac{315}{768} = \frac{105}{256} \\ L_2(0) = \frac{(0+3)(0+1)(0-3)(0-5)(0-7)}{(1+3)(1+1)(1-3)(1-5)(1-7)} = \frac{315}{384} = \frac{105}{128} \end{array} \right\}$$

$$\left. \begin{aligned} L_3(0) &= \frac{(0+3)(0+1)(0-1)(0-5)(0-7)}{(3+3)(3+1)(3-1)(3-5)(3-7)} = -\frac{105}{384} = -\frac{35}{128} \\ L_4(0) &= \frac{(0+3)(0+1)(0-1)(0-3)(0-7)}{(5+3)(5+1)(5-1)(5-3)(5-7)} = \frac{63}{768} = \frac{21}{256} \\ L_5(0) &= \frac{(0+3)(0+1)(0-1)(0-3)(0-5)}{(7+3)(7+1)(7-1)(7-3)(7-5)} = -\frac{45}{3840} = -\frac{3}{256} \end{aligned} \right\}$$

$$p(0) = -14 \cdot \frac{7}{256} + 4 \cdot \frac{105}{256} + 2 \cdot \frac{105}{256} - 8 \cdot \frac{35}{128} + 22 \cdot \frac{21}{256} - 44 \cdot \frac{3}{256}$$

$$p(0) = \frac{-98 + 420 + 210 - 560 + 462 - 132}{256} = \frac{302}{256} = 1.1796875$$

$$\left. \begin{aligned} L_0(2) &= \frac{(2+1)(2-1)(2-3)(2-5)(2-7)}{(-3+1)(-3-1)(-3-3)(-3-5)(-3-7)} = \frac{45}{3840} = \frac{3}{256} \\ L_1(2) &= \frac{(2+3)(2-1)(2-3)(2-5)(2-7)}{(-1+3)(-1-1)(-1-3)(-1-5)(-1-7)} = -\frac{75}{768} = -\frac{25}{256} \\ L_2(2) &= \frac{(2+3)(2+1)(2-3)(2-5)(2-7)}{(1+3)(1+1)(1-3)(1-5)(1-7)} = \frac{225}{384} = \frac{75}{128} \\ L_3(2) &= \frac{(2+3)(2+1)(2-1)(2-5)(2-7)}{(3+3)(3+1)(3-1)(3-5)(3-7)} = \frac{225}{384} = \frac{75}{128} \\ L_4(2) &= \frac{(2+3)(2+1)(2-1)(2-3)(2-7)}{(5+3)(5+1)(5-1)(5-3)(5-7)} = -\frac{75}{768} = -\frac{25}{256} \\ L_5(2) &= \frac{(2+3)(2+1)(2-1)(2-3)(2-5)}{(7+3)(7+1)(7-1)(7-3)(7-5)} = \frac{45}{3840} = \frac{3}{256} \end{aligned} \right\}$$

$$p(2) = 14 \cdot \frac{3}{256} - 4 \cdot \frac{25}{256} + 2 \cdot \frac{75}{128} + 8 \cdot \frac{75}{128} - 22 \cdot \frac{25}{256} + 44 \cdot \frac{3}{256}$$

$$p(2) = \frac{42 - 100 + 300 + 1200 - 550 + 132}{256} = \frac{1024}{256} = 4$$