

Bloque III. Sistemas de ecuaciones

Tema 2 Fundamentos de Determinantes

Ejercicios resueltos

III.2-1 Calcula los determinantes de las siguientes matrices de orden dos:

$$a) A = \begin{pmatrix} -3 & -4 \\ 2 & -3 \end{pmatrix}$$

$$f) A = \begin{pmatrix} -5 & -1 \\ -2 & -3 \end{pmatrix}$$

$$b) A = \begin{pmatrix} -2 & 0 \\ -1 & 5 \end{pmatrix}$$

$$g) A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

$$c) A = \begin{pmatrix} 5 & -1 \\ -2 & 3 \end{pmatrix}$$

$$h) A = \begin{pmatrix} -1 & 0 \\ -5 & 3 \end{pmatrix}$$

$$d) A = \begin{pmatrix} 2 & -3 \\ 5 & -4 \end{pmatrix}$$

$$i) A = \begin{pmatrix} 4 & -2 \\ 4 & 2 \end{pmatrix}$$

$$e) A = \begin{pmatrix} 4 & 1 \\ 8 & 2 \end{pmatrix}$$

$$j) A = \begin{pmatrix} 0 & 3 \\ 0 & 6 \end{pmatrix}$$

Solución

$$a) A = \begin{pmatrix} -3 & -4 \\ 2 & -3 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -3 & -4 \\ 2 & -3 \end{vmatrix} = (-3) \cdot (-3) - [(-4) \cdot (2)] = 9 - (-8) = 9 + 8 = 17$$

$$b) A = \begin{pmatrix} -2 & 0 \\ -1 & 5 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -2 & 0 \\ -1 & 5 \end{vmatrix} = (-2) \cdot (5) - [(0) \cdot (-1)] = -10 - (0) = -10$$

$$c) A = \begin{pmatrix} 5 & -1 \\ -2 & 3 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix} = (5) \cdot (3) - [(-1) \cdot (-2)] = 15 - (2) = 13$$

$$d) A = \begin{pmatrix} 2 & -3 \\ 5 & -4 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 2 & -3 \\ 5 & -4 \end{vmatrix} = (2) \cdot (-4) - [(-3) \cdot (5)] = -8 - (-15) = -8 + 15 = 7$$

$$e) A = \begin{pmatrix} 4 & 1 \\ 8 & 2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 4 & 1 \\ 8 & 2 \end{vmatrix} = (4) \cdot (2) - [(1) \cdot (8)] = 8 - (8) = 0$$

$$f) A = \begin{pmatrix} -5 & -1 \\ -2 & -3 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -5 & -1 \\ -2 & -3 \end{vmatrix} = (-5) \cdot (-3) - [(-1) \cdot (-2)] = 15 - (2) = 13$$

$$g) A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = (2) \cdot (3) - [(0) \cdot (0)] = 6 - 0 = 6$$

$$h) A = \begin{pmatrix} -1 & 0 \\ -5 & 3 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -1 & 0 \\ -5 & 3 \end{vmatrix} = (-1) \cdot (3) - [(0) \cdot (-5)] = -3 - 0 = -3$$

$$i) A = \begin{pmatrix} 4 & -2 \\ 4 & 2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 4 & -2 \\ 4 & 2 \end{vmatrix} = (4) \cdot (2) - [(-2) \cdot (4)] = 8 - (-8) = 8 + 8 = 16$$

$$j) A = \begin{pmatrix} 0 & 3 \\ 0 & 6 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 0 & 3 \\ 0 & 6 \end{vmatrix} = (0) \cdot (6) - [(3) \cdot (0)] = 0 - (0) = 0$$

III.2-2 Calcula los determinantes de las siguientes matrices de orden tres utilizando la regla de Sarrus:

$$a) A = \begin{pmatrix} 3 & 2 & 1 \\ 5 & -2 & -3 \\ -2 & 4 & 4 \end{pmatrix}$$

$$e) A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$b) A = \begin{pmatrix} -1 & 2 & 0 \\ 0 & 1 & -2 \\ 3 & 5 & 3 \end{pmatrix}$$

$$f) A = \begin{pmatrix} -1 & 3 & -2 \\ 0 & -1 & 2 \\ 1 & 4 & 2 \end{pmatrix}$$

$$c) A = \begin{pmatrix} 4 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & -1 & 4 \end{pmatrix}$$

$$g) A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & -2 & 5 \end{pmatrix}$$

$$d) A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & -3 & -2 \\ 0 & 4 & 1 \end{pmatrix}$$

$$h) A = \begin{pmatrix} 0 & 1 & -5 \\ 0 & 2 & -7 \\ 0 & 3 & 6 \end{pmatrix}$$

Solución

$$a) A = \begin{pmatrix} 3 & 2 & 1 \\ 5 & -2 & -3 \\ -2 & 4 & 4 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 5 & -2 & -3 \\ -2 & 4 & 4 \end{vmatrix} = [(3) \cdot (-2) \cdot (4) + (5) \cdot (4) \cdot (1) + (-3) \cdot (-2) \cdot (2)] - \\ - [(1) \cdot (-2) \cdot (-2) + (4) \cdot (-3) \cdot (3) + (5) \cdot (2) \cdot (4)]$$

$$|A| = [-24 + 20 + 12] - [4 - 36 + 40] = 8 - 8 = 0$$

$$b) A = \begin{pmatrix} -1 & 2 & 0 \\ 0 & 1 & -2 \\ 3 & 5 & 3 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -1 & 2 & 0 \\ 0 & 1 & -2 \\ 3 & 5 & 3 \end{vmatrix} = [(-1) \cdot (1) \cdot (3) + (0) \cdot (0) \cdot (5) + (-2) \cdot (2) \cdot (3)] - \\ - [(0) \cdot (1) \cdot (3) + (5) \cdot (-2) \cdot (-1) + (2) \cdot (0) \cdot (3)]$$

$$|A| = [-3 + 0 - 12] - [0 + 10 + 0] = -15 - 10 = -25$$

$$c) A = \begin{pmatrix} 4 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & -1 & 4 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 4 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & -1 & 4 \end{vmatrix} = [(4) \cdot (0) \cdot (4) + (-1) \cdot (2) \cdot (3) + (1) \cdot (1) \cdot (2)] - \\ - [(2) \cdot (0) \cdot (2) + (-1) \cdot (1) \cdot (4) + (1) \cdot (3) \cdot (4)]$$

$$|A| = [0 - 6 + 2] - [0 - 4 + 12] = -4 - 8 = -12$$

$$d) A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & -3 & -2 \\ 0 & 4 & 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 0 \\ -1 & -3 & -2 \\ 0 & 4 & 1 \end{vmatrix} = [(1) \cdot (-3) \cdot (1) + (4) \cdot (-1) \cdot (0) + (2) \cdot (-2) \cdot (0)] - \\ - [(0) \cdot (-3) \cdot (0) + (4) \cdot (-2) \cdot (1) + (-1) \cdot (2) \cdot (1)]$$

$$|A| = [-3 + 0 + 0] - [0 - 8 - 2] = -3 - (-10) = -3 + 10 = 7$$

$$e) A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{vmatrix} = [(1) \cdot (0) \cdot (0) + (1) \cdot (2) \cdot (3) + (1) \cdot (1) \cdot (2)] - \\ - [(2) \cdot (0) \cdot (2) + (1) \cdot (1) \cdot (1) + (3) \cdot (1) \cdot (0)]$$

$$|A| = [0 + 6 + 2] - [0 + 1 + 0] = 8 - 1 = 7$$

$$f) A = \begin{pmatrix} -1 & 3 & -2 \\ 0 & -1 & 2 \\ 1 & 4 & 2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -1 & 3 & -2 \\ 0 & -1 & 2 \\ 1 & 4 & 2 \end{vmatrix} = [(-1) \cdot (-1) \cdot (2) + (4) \cdot (0) \cdot (-2) + (3) \cdot (2) \cdot (1)] - \\ - [(-2) \cdot (-1) \cdot (1) + (4) \cdot (2) \cdot (-1) + (0) \cdot (2) \cdot (3)]$$

$$|A| = [2 + 0 + 6] - [2 - 8 + 0] = 8 - (-6) = 8 + 6 = 14$$

$$g) A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & -2 & 5 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & -2 & 5 \end{vmatrix} = [(1) \cdot (4) \cdot (5) + (-2) \cdot (2) \cdot (3) + (6) \cdot (2) \cdot (-1)] - \\ - [(3) \cdot (4) \cdot (-1) + (-2) \cdot (6) \cdot (1) + (2) \cdot (2) \cdot (5)]$$

$$|A| = [20 - 12 - 12] - [-12 - 12 + 20] = -4 - (-4) = -4 + 4 = 0$$

$$h) A = \begin{pmatrix} 0 & 1 & -5 \\ 0 & 2 & -7 \\ 0 & 3 & 6 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 0 & 1 & -5 \\ 0 & 2 & -7 \\ 0 & 3 & 6 \end{vmatrix} = [(0) \cdot (2) \cdot (6) + (3) \cdot (0) \cdot (-5) + (-7) \cdot (1) \cdot (0)] - \\ - [(-5) \cdot (2) \cdot (0) + (3) \cdot (-7) \cdot (0) + (6) \cdot (0) \cdot (1)]$$

$$|A| = [0 + 0 + 0] - [0 + 0 + 0] = 0 - 0 = 0$$

III.2-3 Calcula los de terminantes de las siguientes matrices desarrollando por una fila o columna:

$$a) A = \begin{pmatrix} 1 & 3 & 2 \\ 6 & 5 & 4 \\ 9 & 7 & 8 \end{pmatrix}$$

$$b) A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 9 & -1 & 0 \end{pmatrix}$$

$$c) A = \begin{pmatrix} 1 & 3 & 1 \\ 5 & 4 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

$$d) A = \begin{pmatrix} 3 & -2 & 1 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$$

$$e) A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 3 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ -1 & 2 & 3 & 1 \end{pmatrix}$$

$$f) A = \begin{pmatrix} -1 & 3 & 0 & -2 \\ 0 & -2 & 3 & 2 \\ 1 & 0 & 4 & 2 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$g) A = \begin{pmatrix} -1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 3 & -2 & 0 & 2 \end{pmatrix}$$

$$h) A = \begin{pmatrix} 3 & -1 & 2 & 4 \\ -1 & 1 & 0 & -3 \\ -5 & 4 & 1 & 2 \\ 2 & -2 & 0 & 6 \end{pmatrix}$$

Solución

$$a) A = \begin{pmatrix} 1 & 3 & 2 \\ 6 & 5 & 4 \\ 9 & 7 & 8 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 2 \\ 6 & 5 & 4 \\ 9 & 7 & 8 \end{vmatrix} \stackrel{1^{\text{a}} \text{ fila}}{=} (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} 5 & 4 \\ 7 & 8 \end{vmatrix} + (-1)^{1+2} \cdot 3 \cdot \begin{vmatrix} 6 & 4 \\ 9 & 8 \end{vmatrix} + (-1)^{1+3} \cdot 2 \cdot \begin{vmatrix} 6 & 5 \\ 9 & 7 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 5 & 4 \\ 7 & 8 \end{vmatrix} - 3 \cdot \begin{vmatrix} 6 & 4 \\ 9 & 8 \end{vmatrix} + 2 \cdot \begin{vmatrix} 6 & 5 \\ 9 & 7 \end{vmatrix} = [40 - 28] - 3 \cdot [48 - 36] + 2 \cdot [42 - 45]$$

$$|A| = 12 - 36 - 6 = -30$$

$$b) A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 9 & -1 & 0 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 9 & -1 & 0 \end{vmatrix} \stackrel{3^{\text{a}} \text{ columna}}{=} (-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} 3 & 1 \\ 9 & -1 \end{vmatrix} + (-1)^{2+3} \cdot 0 \cdot A_{23} + (-1)^{3+3} \cdot 0 \cdot A_{33}$$

$$|A| = \begin{vmatrix} 3 & 1 \\ 9 & -1 \end{vmatrix} = [-3 - 9] = -12$$

$$c) A = \begin{pmatrix} 1 & 3 & 1 \\ 5 & 4 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 1 \\ 5 & 4 & 2 \\ 1 & 0 & 1 \end{vmatrix} \stackrel{3^{\text{a}} \text{ fila}}{=} (-1)^{3+1} \cdot 1 \cdot \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} + (-1)^{3+2} \cdot 0 \cdot A_{32} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix} = [6 - 4] + [4 - 15] = 2 - 11 = -9$$

$$d) A = \begin{pmatrix} 3 & -2 & 1 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{vmatrix} \stackrel{3^{\text{a}} \text{ fila}}{=} (-1)^{3+1} \cdot 0 \cdot A_{31} + (-1)^{3+2} \cdot 2 \cdot \begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} + (-1)^{3+3} \cdot 0 \cdot A_{33}$$

$$|A| = -2 \cdot \begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} = -2 \cdot [3 - 3] = 0$$

$$e) \mathbf{A} = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 3 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ -1 & 2 & 3 & 1 \end{pmatrix}$$

$$|\mathbf{A}| = \begin{vmatrix} 1 & 2 & 1 & 2 \\ 3 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ -1 & 2 & 3 & 1 \end{vmatrix} \stackrel{3^a \text{ fila}}{=} (-1)^{3+1} \cdot 0 \cdot \mathbf{A}_{31} + (-1)^{3+2} \cdot 2 \cdot \begin{vmatrix} 1 & 1 & 2 \\ 3 & 0 & 1 \\ -1 & 3 & 1 \end{vmatrix} +$$

$$+ (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} 1 & 2 & 2 \\ 3 & 0 & 1 \\ -1 & 2 & 1 \end{vmatrix} + (-1)^{3+4} \cdot 0 \cdot \mathbf{A}_{34}$$

$$|\mathbf{A}| = -2 \cdot \begin{vmatrix} 1 & 1 & 2 \\ 3 & 0 & 1 \\ -1 & 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 2 \\ 3 & 0 & 1 \\ -1 & 2 & 1 \end{vmatrix} = -2 \cdot \{[0+18-1] - [0+3+3]\} +$$

$$+ \{[0+12-2] - [0+2+6]\} = -2 \cdot (11) + 2 = -22 + 2 = -20$$

$$f) \mathbf{A} = \begin{pmatrix} -1 & 3 & 0 & -2 \\ 0 & -2 & 3 & 2 \\ 1 & 0 & 4 & 2 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$|\mathbf{A}| = \begin{vmatrix} -1 & 3 & 0 & -2 \\ 0 & -2 & 3 & 2 \\ 1 & 0 & 4 & 2 \\ 0 & 0 & 0 & -1 \end{vmatrix} \stackrel{4^a \text{ fila}}{=} (-1)^{4+1} \cdot 0 \cdot \mathbf{A}_{41} + (-1)^{4+2} \cdot 0 \cdot \mathbf{A}_{42} +$$

$$+ (-1)^{4+3} \cdot 0 \cdot \mathbf{A}_{43} + (-1)^{4+4} \cdot (-1) \cdot \begin{vmatrix} -1 & 3 & 0 \\ 0 & -2 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

$$|\mathbf{A}| = - \begin{vmatrix} -1 & 3 & 0 \\ 0 & -2 & 3 \\ 1 & 0 & 4 \end{vmatrix} = - \{[8+0+9] - [0+0+0]\} = -17$$

$$g) A = \begin{pmatrix} -1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 3 & -2 & 0 & 2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 3 & -2 & 0 & 2 \end{vmatrix} \stackrel{2^{\text{a columna}}}{=} (-1)^{1+2} \cdot 0 \cdot A_{12} + (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -1 & -1 & 1 \\ 0 & 1 & 1 \\ 3 & 0 & 2 \end{vmatrix} +$$

$$+ (-1)^{3+2} \cdot 0 \cdot A_{32} + (-1)^{4+2} \cdot (-2) \cdot \begin{vmatrix} -1 & -1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$|A| = \begin{vmatrix} -1 & -1 & 1 \\ 0 & 1 & 1 \\ 3 & 0 & 2 \end{vmatrix} - 2 \cdot \begin{vmatrix} -1 & -1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$|A| = [(-2-3)-(3+0+0)] - 2 \cdot [(0+2+0)-(0-1-2)] = -8-10 = -18$$

$$h) A = \begin{pmatrix} 3 & -1 & 2 & 4 \\ -1 & 1 & 0 & -3 \\ -5 & 4 & 1 & 2 \\ 2 & -2 & 0 & 6 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 3 & -1 & 2 & 4 \\ -1 & 1 & 0 & -3 \\ -5 & 4 & 1 & 2 \\ 2 & -2 & 0 & 6 \end{vmatrix} \stackrel{3^{\text{a columna}}}{=} (-1)^{1+3} \cdot 2 \cdot \begin{vmatrix} -1 & 1 & -3 \\ -5 & 4 & 2 \\ 2 & -2 & 6 \end{vmatrix} + (-1)^{2+3} \cdot 0 \cdot A_{23} +$$

$$+ (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} 3 & -1 & 4 \\ -1 & 1 & -3 \\ 2 & -2 & 6 \end{vmatrix} + (-1)^{4+3} \cdot 0 \cdot A_{43}$$

$$|A| = 2 \cdot \begin{vmatrix} -1 & 1 & -3 \\ -5 & 4 & 2 \\ 2 & -2 & 6 \end{vmatrix} + \begin{vmatrix} 3 & -1 & 4 \\ -1 & 1 & -3 \\ 2 & -2 & 6 \end{vmatrix}$$

$$|A| = 2 \cdot [(-24-30+4)-(-24+4-30)] + [(18+8+6)-(8+18+6)] = 0$$

III.2-4 Calcula la inversa por determinantes de las siguientes matrices:

$$a) A = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix} \quad b) A = \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix} \quad c) A = \begin{pmatrix} 0 & -1 & 3 \\ -2 & 3 & 0 \\ -5 & -3 & 2 \end{pmatrix}$$

Solución

$$a) A = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix} \Rightarrow |A| = \begin{vmatrix} 3 & 4 \\ 5 & 2 \end{vmatrix} = 6 - 20 = -14$$

$$\left. \begin{array}{l} A_{11} = |2| = 2 \quad A_{21} = -|4| = -4 \\ A_{12} = -|5| = -5 \quad A_{22} = |3| = 3 \end{array} \right\} \Rightarrow \text{Adj}(A^T) = \begin{pmatrix} 2 & -4 \\ -5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}(A^T) = \frac{1}{-14} \cdot \begin{pmatrix} 2 & -4 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} -1/7 & 2/7 \\ 5/14 & -3/14 \end{pmatrix}$$

$$b) A = \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix} \Rightarrow |A| = \begin{vmatrix} -1 & 2 \\ -3 & 4 \end{vmatrix} = -4 + 6 = 2$$

$$\left. \begin{array}{l} A_{11} = |2| = 4 \quad A_{21} = -|2| = -2 \\ A_{12} = -|-3| = 3 \quad A_{22} = |-1| = -1 \end{array} \right\} \Rightarrow \text{Adj}(A^T) = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}(A^T) = \frac{1}{2} \cdot \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3/2 & -1/2 \end{pmatrix}$$

$$c) A = \begin{pmatrix} 0 & -1 & 3 \\ -2 & 3 & 0 \\ -5 & -3 & 2 \end{pmatrix} \Rightarrow |A| = \begin{vmatrix} 0 & -1 & 3 \\ -2 & 3 & 0 \\ -5 & -3 & 2 \end{vmatrix} = 0 + 18 + 0 - (-45 + 0 + 4) = 59$$

$$\left. \begin{array}{l} A_{11} = \begin{vmatrix} 3 & 0 \\ -3 & 2 \end{vmatrix} = 6 \quad A_{21} = -\begin{vmatrix} -1 & 3 \\ -3 & 2 \end{vmatrix} = -7 \quad A_{31} = \begin{vmatrix} -1 & 3 \\ 3 & 0 \end{vmatrix} = -9 \\ A_{12} = -\begin{vmatrix} -2 & 0 \\ -5 & 2 \end{vmatrix} = 4 \quad A_{22} = \begin{vmatrix} 0 & 3 \\ -5 & 2 \end{vmatrix} = 15 \quad A_{32} = -\begin{vmatrix} 0 & 3 \\ -2 & 0 \end{vmatrix} = -6 \\ A_{13} = \begin{vmatrix} -2 & 3 \\ -5 & -3 \end{vmatrix} = 21 \quad A_{23} = -\begin{vmatrix} 0 & -1 \\ -5 & -3 \end{vmatrix} = 5 \quad A_{33} = \begin{vmatrix} 0 & -1 \\ -2 & 3 \end{vmatrix} = -2 \end{array} \right\} \Rightarrow \text{Adj}(A^T) = \begin{pmatrix} 6 & -7 & -9 \\ 4 & 15 & -6 \\ 21 & 5 & -2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}(A^T) = \frac{1}{59} \cdot \begin{pmatrix} 6 & -7 & -9 \\ 4 & 15 & -6 \\ 21 & 5 & -2 \end{pmatrix}$$