Multirobot Systems

Lecture
Multirobot formation control

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Departamento de Informática e Ingeniería de Sistemas
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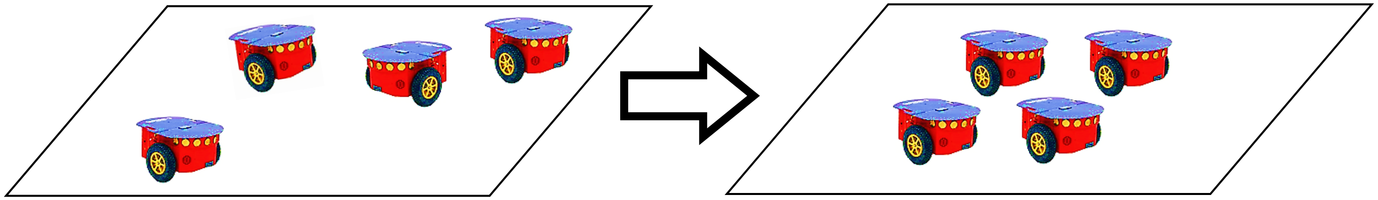
Introduction

- Multiagent scenarios => Multiagent formations
  - A team of mobile agents capable of
    - Autonomous perception, localization, and navigation
  - Applications with richer task specification:
    - Behavior defined relative to the group
      - Environment surveillance
      - Mapping
      - Exploration
      - Search and rescue missions, etc.
  - Collective motion tasks:
    - Formation control, rendezvous, flocking, etc.
    - Example: Formation shape stabilization



Introduction

- Different possible architectures, associated with different topologies
  - How many sensors?
  - Who carries them?
  - How does the information enter the control loop?
- Requirements for global reference frames is often difficult to ensure
  - GPS (Global Positioning System)
  - Optical Motion Capture System

State of the art

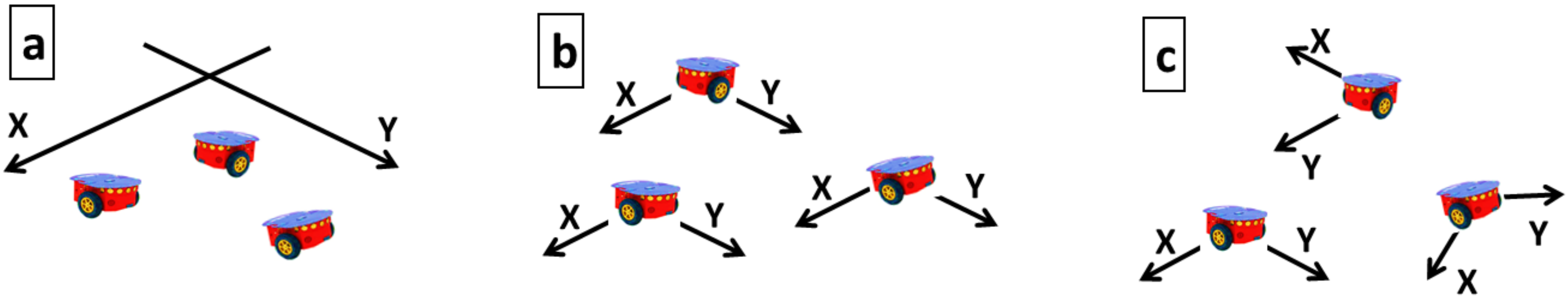
- Multiagent formation control
  - Absolute agent positions
    - M. M. Zavlanos and G. J. Pappas, "Distributed formation control with permutation symmetries," in IEEE Conference on Decision and Control, 2007, pp. 2894–2899.
  - Relative interagent distances
    - K.-K. Oh and H.-S. Ahn, "Formation control of mobile agents based on inter-agent distance dynamics," Automatica, vol. 47, no. 10, pp. 2306 – 2312, 2011.
  - Relative interagent positions
    - H. Tanner and A. Boddu, "Multiagent navigation functions revisited," IEEE Trans. on Robotics, vol. 28, no. 6, pp. 1346–1359, 2012
  - Common orientation reference
    - K.-K. Oh and H.-S. Ahn, "Formation control and network localization via orientation alignment," IEEE Transactions on Automatic Control, vol. 59 (2), pp. 540–545, 2014.

State of the art

- Multiagent formation control
  - Leader-follower formation
    - J. Chen, D. Sun, J. Yang, and H. Chen, “Leader-follower formation control of multiple non-holonomic mobile robots incorporating a receding-horizon scheme,” International Journal of Robotics Research, vol. 29, no. 6, pp. 727–747, 2010
  - Circular formation
    - N. Moshtagh, N. Michael, A. Jadbabaie, and K. Daniilidis, “Vision-based, distributed control laws for motion coordination of nonholonomic robots,” IEEE Trans. Rob., vol. 25, no. 4, pp. 851–860, 2009
  - 3D formation control
    - M. Turpin, N. Michael, and V. Kumar, “Decentralized formation control with variable shapes for aerial robots,” in IEEE International Conference on Robotics and Automation, 2012, pp. 23–30

Problem definition

- Coordinate-free formation control
  - Scenarios for decentralized formation control

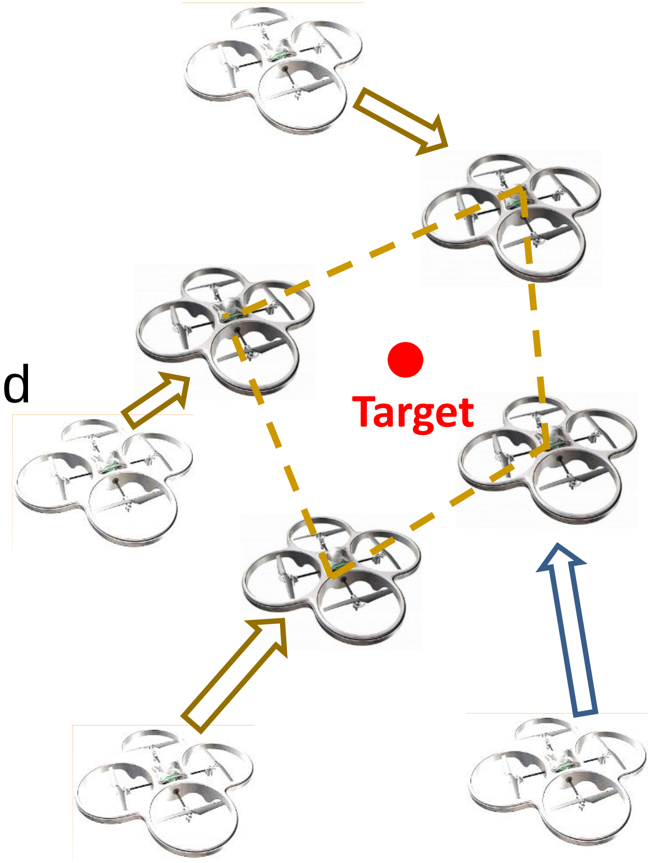


  - (a) and (b) require global references (by e.g. GPS)
  - Only (c) is coordinate-free => amenable, e.g. to a vision-based implementation

  - Issue: formation stabilization in (c) generally:
    - Requires leader robots, or
    - Is only locally stable (distance-based formation control)

Problem definition

- Problem: **<u>3D target enclosing</u>** with a UAV team
- Applications:
  - Escorting
  - Entrapment of an unfriendly element
  - Collective perception
- Task: make a team of UAVs form a desired geometric pattern with the target at its centroid



- Each UAV uses the locally measured relative positions of the other UAVs and the target, without a global reference frame (the target should be the only "reference" in this task)
- Any three-dimensional pattern is possible (improves flexibility, quality of perception and size of escape areas with respect to planar ones)

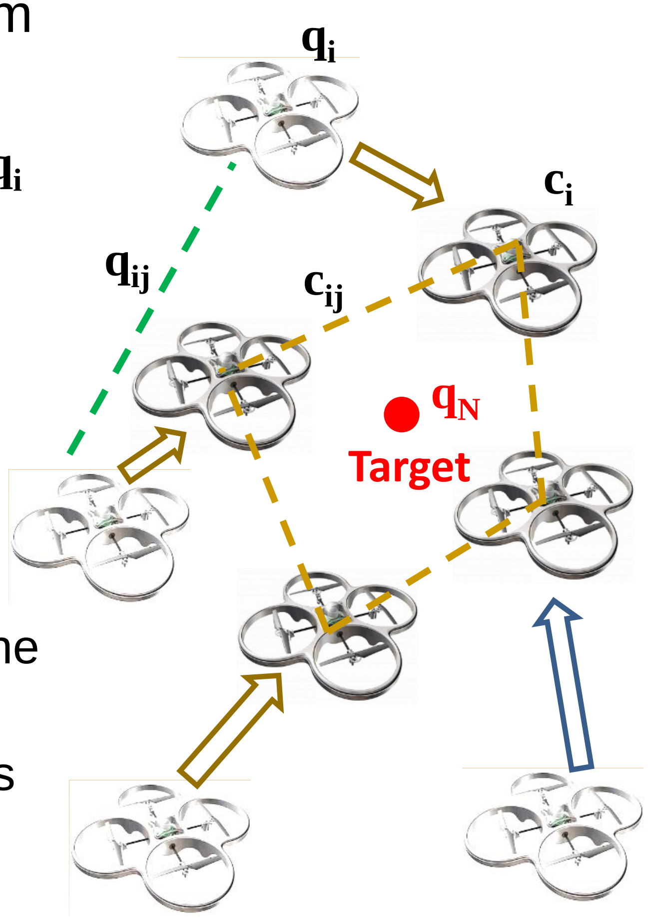
Problem definition

- Problem: 3D target enclosing with a UAV team
- Definitions:
  - Consider N-1 robots in 3D space with position $\mathbf{q_i}$
  - Dynamics: single integrator model $\dot{\mathbf{q}}_i = \mathbf{u}_i$
    - Position vector of robot $\mathbf{i}$ is $\mathbf{q_i}$
    - Its control input $\mathbf{u_i}$
  - Position of the target to be enclosed: $\mathbf{q_N}$
  - Desired enclosing configuration:
    - Inter-robot relative position vectors: $\mathbf{c_{ij}}$
    - Desired vectors from the target to each of the N-1 robots: $\mathbf{c_{Ni}}$
    - Consider the desired position of the target is the centroid of the desired configuration

$$\sum_{i \in 1,\dots,N-1} \mathbf{c}_{Ni} = 0$$

  - Current relative position vectors: $\mathbf{q_{ij}}$

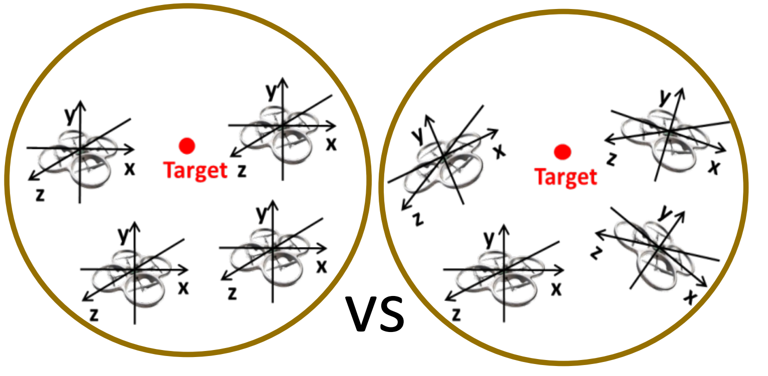
$$\mathbf{q}_{ij} = \mathbf{q}_i - \mathbf{q}_j$$



Enclosing control law

- Problem: 3D target enclosing with a UAV team
- Goal: Achieve formation shape with arbitrary translation and rotation
  - Rotation matrix, required due to the lack of a common orientation reference: $\mathbf{R} \in SO(3)$

$$\mathbf{q_{ji}} = \mathbf{R}\mathbf{c_{ji}}, \quad \forall i, j = 1, \ldots, N$$



  - Since the robots' frames are not equally oriented, we introduce rotation matrices that we define as minimizers of the cost function

$$\gamma = \sum_i \sum_j ||\mathbf{q_{ij}} - \mathbf{R}\mathbf{c_{ij}}||_F^2$$

  - How to compute $\mathbf{R}$ that minimizes the cost function?
    - The Procrustes problem

$\mathbf{R}$?

Enclosing control law

- The Procrustes problem
  - The problem is named after the greek Procrustes, from the greek mythology
  - In ancient Greek legend, Procrustes was a tyrant of Attica, whose real name was Polypemon or Damastes. He would invite strangers into his house and force them into a bed; if they were too long and did not fit, he would cut off their legs; if they were too short, he would stretch them until they died. His death was one of Theseus' first heroic deed.

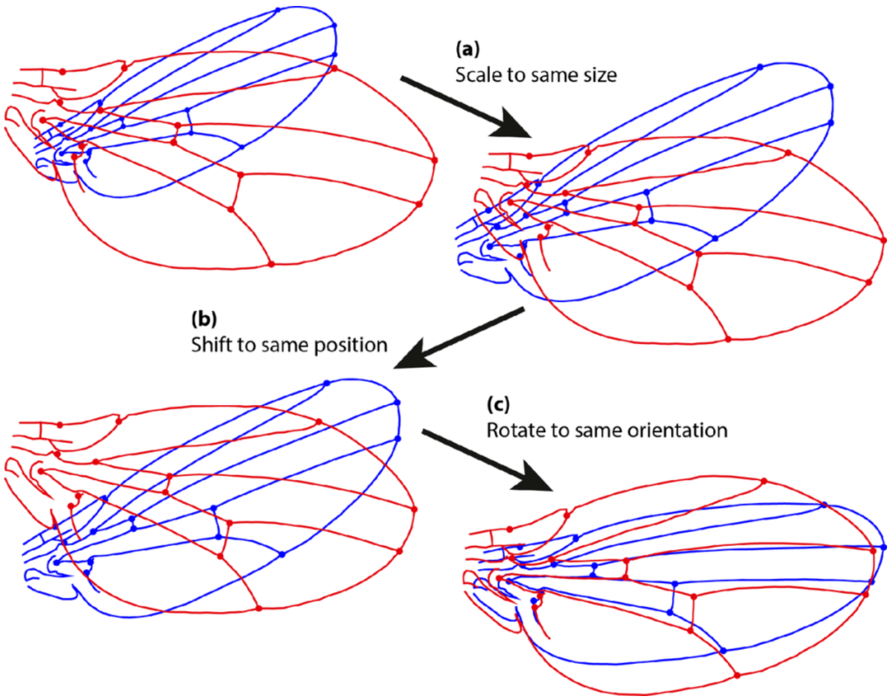
[Gran Enciclopedia del Mundo]

Enclosing control law

- The Procrustes problem
  - The orthogonal Procrustes problem: given two matrices A and B, find an orthogonal matrix Ω which most closely maps A to B:

$$R = \arg\min_{\Omega} \|\Omega A - B\|_F \quad \text{subject to} \quad \Omega^T \Omega = I$$

  - The Kabsch algorithm is a constrained orthogonal Procrustes problem, subject to $\det(\mathbf{R}) = 1$ (where R is a rotation matrix). This method determines the optimal rotation of an object with respect to another.



[https://en.wikipedia.org/wiki/Procrustes_analysis]

Enclosing control law

- Cost function: $\gamma = \sum_i \sum_j ||\mathbf{q_{ij}} - \mathbf{R}\mathbf{c_{ij}}||_F^2$
  - Global information
  - Define **Q**, **C** of size **N*Nx3** $\mathbf{Q} = [\mathbf{q}_{11}...\mathbf{q}_{1N}\ \mathbf{q}_{21}...\mathbf{q}_{2N}...\mathbf{q}_{N1}...\mathbf{q}_{NN}]^T$
    $\mathbf{C} = [\mathbf{c}_{11}...\mathbf{c}_{1N}\ \mathbf{c}_{21}...\mathbf{c}_{2N}...\mathbf{c}_{N1}...\mathbf{c}_{NN}]^T.$
  - $\mathbf{R} \in SO(3)$ that minimizes the cost function:
    - Kabsch algorithm:
      - Singular Value Decomposition (SVD) $\mathbf{A} = \mathbf{C}^T\mathbf{Q} \Rightarrow \mathbf{A} = \mathbf{USV}^T$

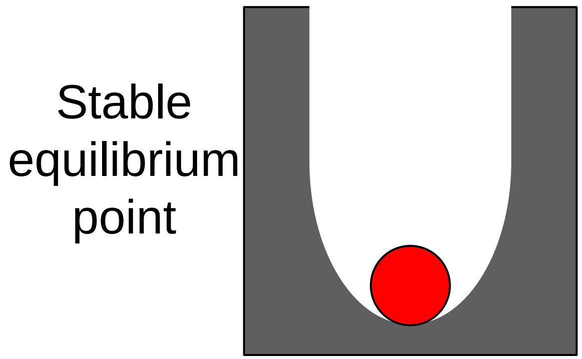
$$\mathbf{R} = \mathbf{VDU}^T = \mathbf{V}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{pmatrix}\mathbf{U}^T \qquad d = sign(det(\mathbf{VU}^T))$$

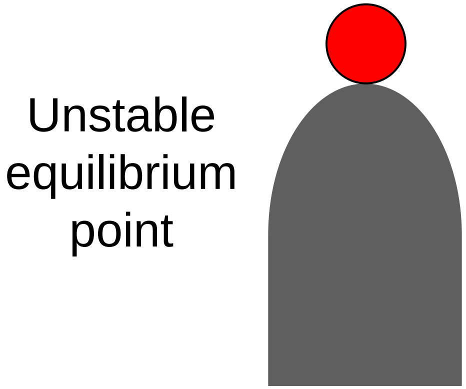
- Control law for each robot, locally computed, for single-integrator kinematics
  - K_c is the control gain
  - **Is this stable?**

$$\dot{\mathbf{q}}_{\mathbf{i}} = K_c(\mathbf{q}_{\mathbf{Ni}} - \mathbf{R}\mathbf{c}_{\mathbf{Ni}})$$

Stability analysis

- It is important that systems are stable. An unstable system is generally useless and potentially dangerous.
- A system is said to be stable if the system being close to its operating point implies that it will always remain around that point.





- Stability of systems
  - Linear Time Invariant systems:
    - Multiple techniques and criteria available (Nyquist, Routh, etc.).
  - Non-linear or time-variant systems:
    - Difficult and sometimes impossible
  - Lyapunov method

Stability analysis

- Lyapunov's methods (1892)
  - For determining the stability of dynamical systems described by ordinary differential equations.
  - First Lyapunov method
    - Applicable when the explicit solution of the differential equations is available.
  - Second Lyapunov method or direct method
    - To analyze the stability without solving the differential equations.

- The basic idea of Lyapunov's direct method is based on the physical fact that if the total "energy" of a mechanical system is continuously dissipated, then, whether the system is linear or nonlinear, it must eventually reach a point of equilibrium (zero energy).
- Therefore, the stability of the system can be analyzed by studying the variation of an energy function associated with the system.

Stability analysis

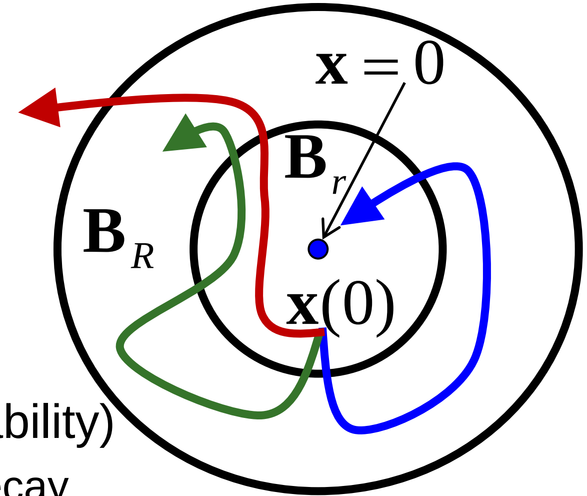
- Lyapunov stability
  - Consider a system with state vector $\mathbf{x}$ such that: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$
  - State or equilibrium point of the system is: $\mathbf{f}(\mathbf{x_e} = \mathbf{0}, t) = \mathbf{0}, \ \forall t$
    - If the system is linear: $\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x}, \ \text{if } \det(\mathbf{A}) \neq 0 \Rightarrow \exists \, \text{only one } \mathbf{x_e}$
    - In nonlinear systems there may be more than one equilibrium point.
  - Lyapunov stability:

$$\forall R > 0, \ \exists r > 0, \ \mathbf{x}(0) \in \mathbf{B}_r \Rightarrow \forall t \geq 0, \ \mathbf{x}(t) \in \mathbf{B}_R$$

    - An equilibrium point is **stable** if for any **R**>**0** there exists an r>0 such that if x(0) belongs to the sphere of radius r, then x(t) will belong to the sphere of radius R for all t.
    - Otherwise the equilibrium point is **unstable**.
  - Asymptotic stability:
    - An equilibrium point x=0 is **asymptotically stable** if it is stable and in addition

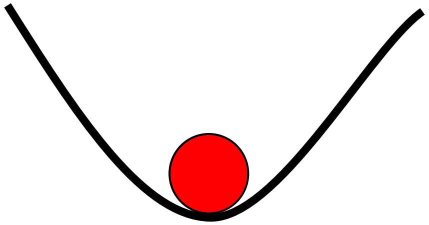
$$\exists r > 0, \mathbf{x}(0) \in \mathbf{B}_r \Rightarrow \mathbf{x}(t) \to 0, t \to \infty$$

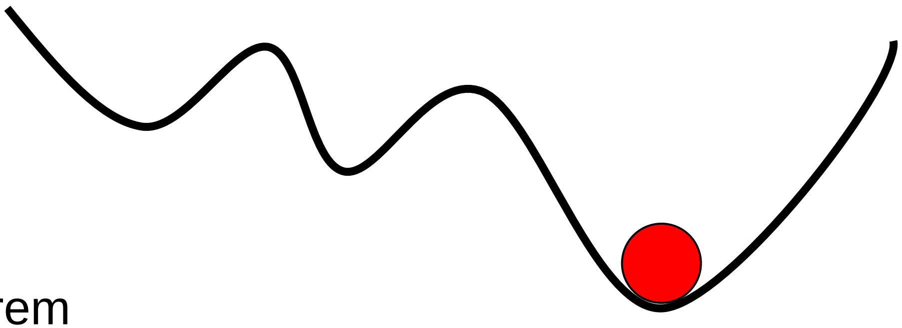
    - Exponential stability (a form of asymptotic stability)
      - The convergence is bounded by exponential decay.



Stability analysis

- Lyapunov stability
  - Global: If asymptotic stability is satisfied for any initial state, the equilibrium point is global and asymptotically stable.
  - Local: If asymptotic stability is not satisfied for every initial state, the equilibrium point is local and asymptotically stable.





  - Lyapunov's direct method theorem

$$\text{Given system } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t), \text{ with } \mathbf{f}(\mathbf{0}, t) = \mathbf{0}, \ \forall t, \ \text{if } \exists V(\mathbf{x}, t) \ni$$

$$V(\mathbf{x}, t) > 0 \text{ (positive definite)}$$

$$\dot{V}(\mathbf{x}, t) < 0 \text{ (negative definite)}$$

$$\Rightarrow \text{The equilibrium point in the origen is asymptotically stable}$$

  - The scalar function V(x, t) is called Lyapunov function.
  - A function V is positive definite if $V(\mathbf{x}) > 0, \forall \mathbf{x} \neq \mathbf{0} \wedge V(\mathbf{0}) = 0$

Stability analysis

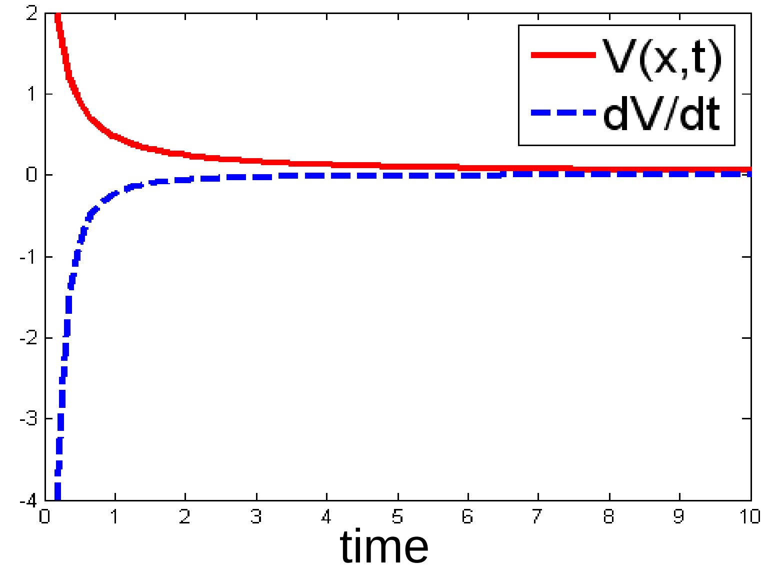
- Stability analysis example

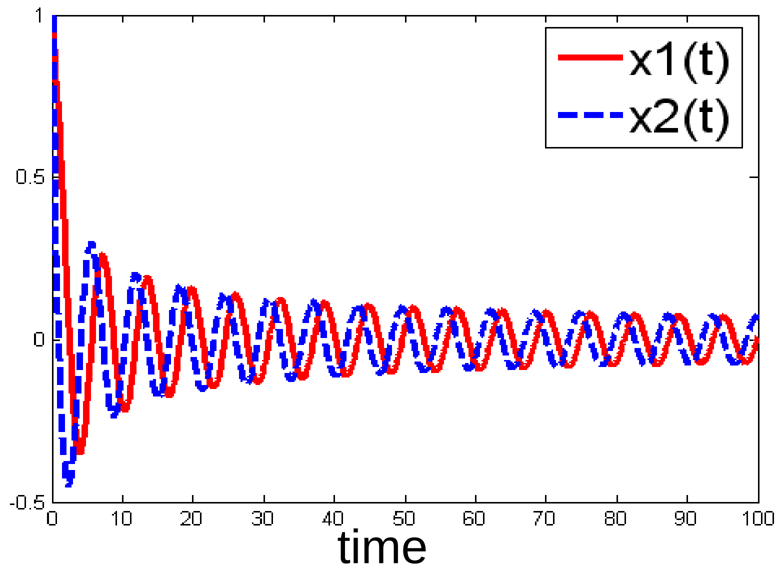
$$\begin{cases} \dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2) \\ \dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2) \end{cases}$$

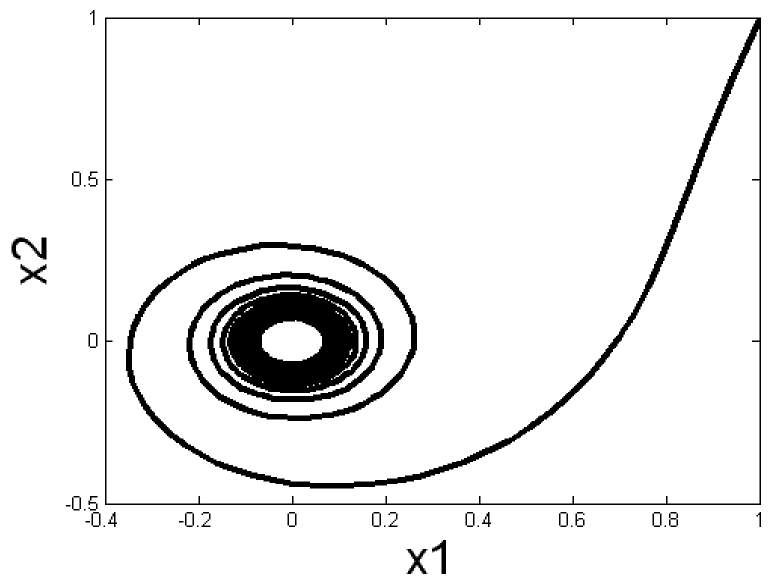
$$V(\mathbf{x}) = x_1^2 + x_2^2 > 0$$

$$\dot{V}(\mathbf{x}) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2$$

$$= -2(x_1^2 + x_2^2)^2 < 0$$







Stability analysis

- Stability of the target enclosing strategy
  - The gradient with respect to the rotation matrix **R** must be null

$$\nabla_{\mathbf{R}}\gamma = \mathbf{R}^T\mathbf{A}^T - \mathbf{A}\mathbf{R} = \mathbf{0}$$

  - From the control law: $\dot{\mathbf{q}}_{\mathbf{i}} = K_c(\mathbf{q}_{\mathbf{Ni}} - \mathbf{R}\mathbf{c}_{\mathbf{Ni}})$
  - We obtain $\dot{\mathbf{q}}_{\mathbf{ij}}(t) = \dot{\mathbf{q}}_{\mathbf{i}}(t) - \dot{\mathbf{q}}_{\mathbf{j}}(t) = -K_c[\mathbf{q}_{\mathbf{ij}}(t) - \mathbf{R}(t)\mathbf{c}_{\mathbf{ij}}]$
  - After some manipulation we get

$$\dot{\mathbf{R}}^T\mathbf{A}^T - \mathbf{A}\dot{\mathbf{R}} = \mathbf{0} \Longrightarrow \boxed{\dot{\mathbf{R}} = \mathbf{0}}$$

$$\boxed{\dot{\mathbf{q}}_{\mathbf{ij}}(t) = -K_c[\mathbf{q}_{\mathbf{ij}}(t) - \mathbf{R}(t)\mathbf{c}_{\mathbf{ij}}] = -K_c[\mathbf{q}_{\mathbf{ij}}(t) - \mathbf{R}_0\mathbf{c}_{\mathbf{ij}}]}$$

  - Therefore, the system **converges exponentially** to the desired configuration with the target in the centroid of the attained formation
- Globally convergent relative position-based formation stabilization in the absence of global reference frames or leader robots

Formation control

- The enclosing control law can be used for standard formation stabilization tasks
- Consider only in the control the robot-to-robot vectors:

$$\dot{\mathbf{q}}_{\mathbf{i}} = \frac{K_c}{N}\left[\sum_j \mathbf{q}_{\mathbf{ji}} + \mathbf{q}_{\mathbf{Nj}} - \mathbf{R}\left(\sum_j \mathbf{c}_{\mathbf{ji}} + \mathbf{c}_{\mathbf{Nj}}\right)\right] \quad j = 1, \ldots, N$$

- Keeping only the inter-robot vectors measured by i, we can define the following control law:

$$\dot{\mathbf{q}}_{\mathbf{i}} = K\left[\sum_j \mathbf{q}_{\mathbf{ji}} - \mathbf{R}\sum_j \mathbf{c}_{\mathbf{ji}}\right] \qquad j = 1, \ldots, N-1$$

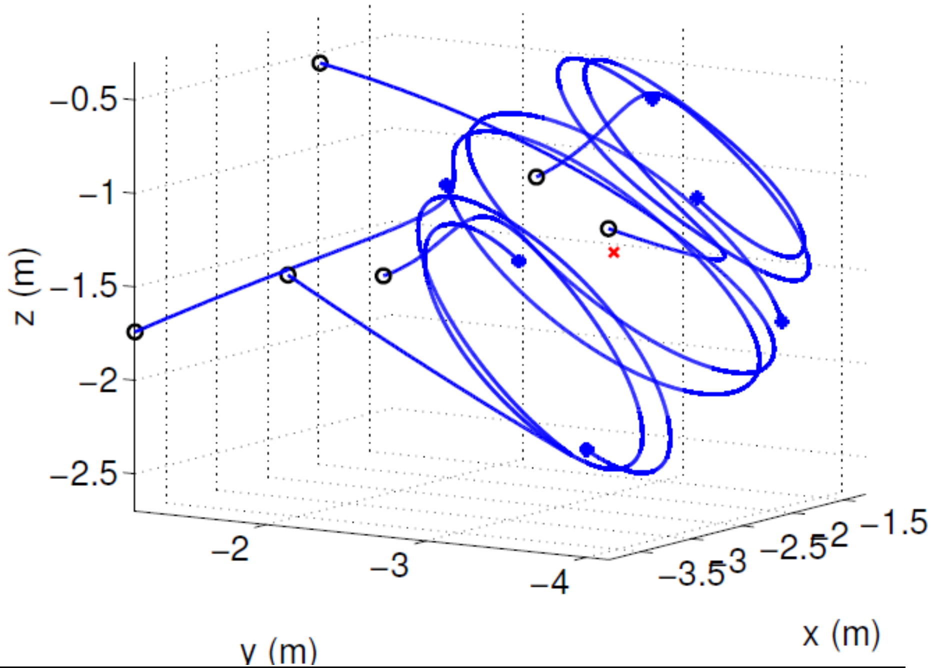
- This control law stabilizes the UAVs to a formation specified by $\mathbf{c}_{\mathbf{ji}}$

Orbiting control

- Maintain the enclosing of the target while the robots gyrate around it
  - Enclosing control law: $\dot{\mathbf{q}}_{\mathbf{i}} = K_c(\mathbf{q}_{\mathbf{Ni}} - \mathbf{R}\mathbf{c}_{\mathbf{Ni}})$
- By means of additive velocity component proportional to the relative vector from robot i to i+1

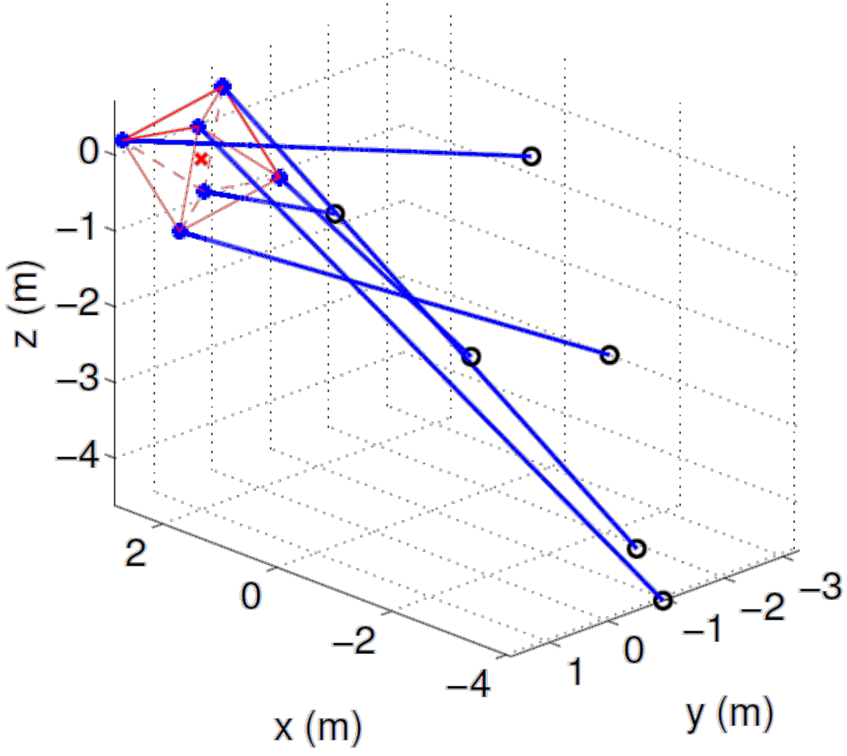
$$\dot{\mathbf{q}}_{\mathbf{i}} = K_c(\mathbf{q}_{\mathbf{Ni}} - \mathbf{R}\mathbf{c}_{\mathbf{Ni}}) - K_g(\mathbf{q}_{\mathbf{i+1}} - \mathbf{q}_{\mathbf{i}})$$

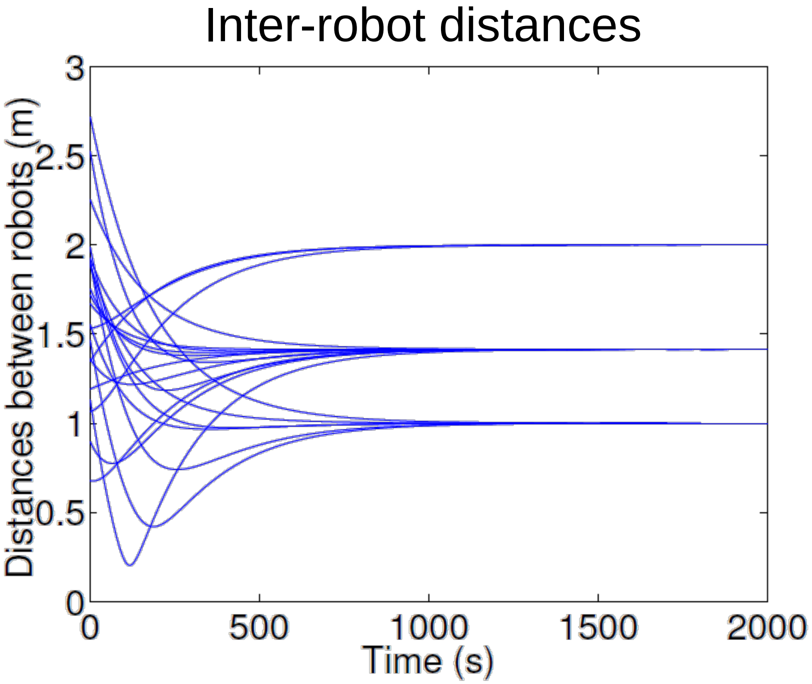
- This is a cyclic pursuit strategy



Simulations

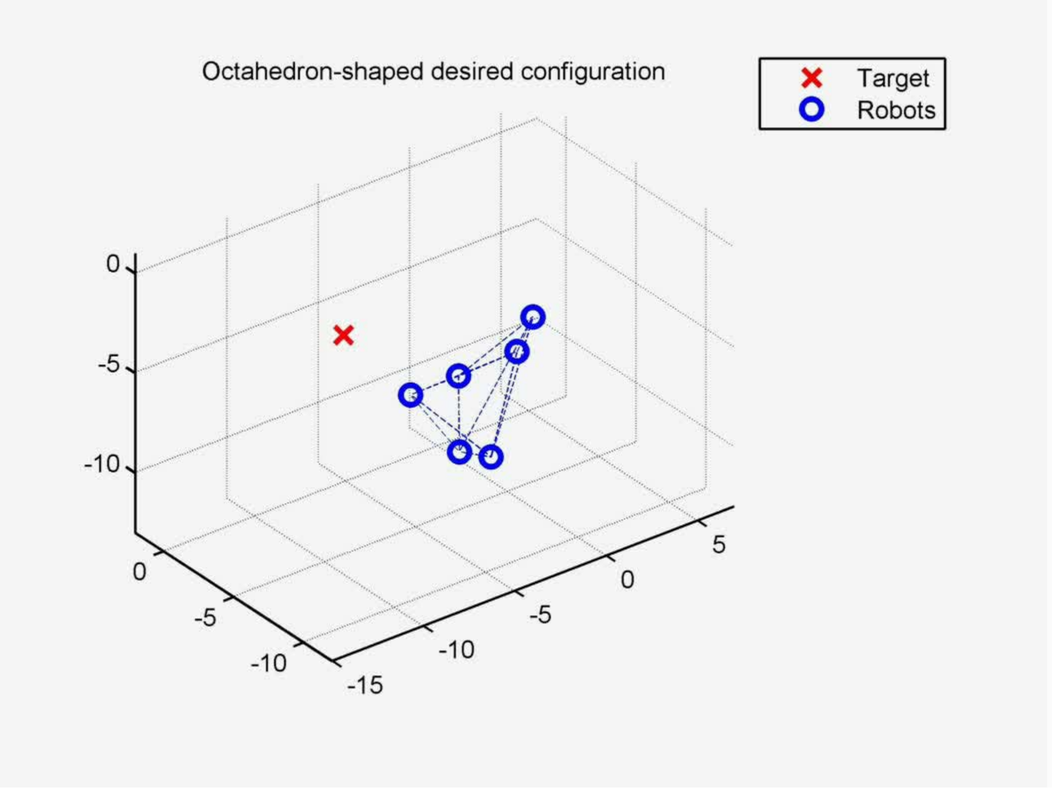
- Robot paths from arbitrary initial positions (circles) to the positions in an octahedron-shaped enclosing formation (stars) of a target (cross) situated at coordinates (2,1,0).





Simulations

- Simulation examples
  - Efficient trajectories
  - Arbitrary geometries
  - Moving target and gyrating formation



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