



# Multirobot Systems

## Lecture Persistent coverage

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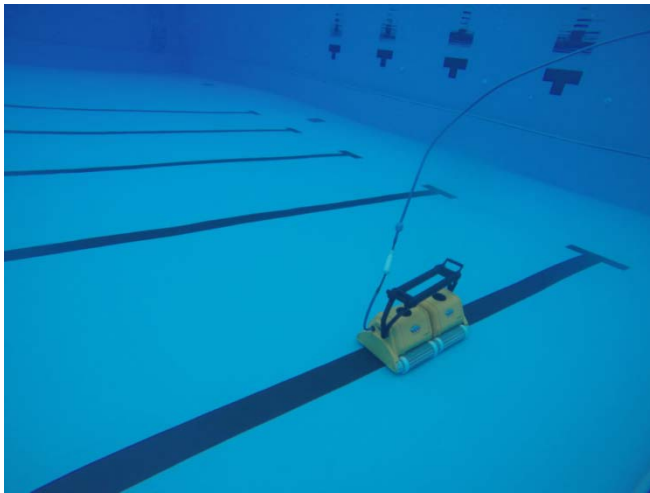
# Introduction

## ■ What is coverage?

- Coverage of an area refers to the goal of collecting data or performing some action at each point in a domain of interest
- Some applications:
  - Cleaning
  - Pool cleaner
  - Lawn mower
  - Snow-blower



[Roomba robot]



[<http://roboticpoolcleanersingapore.com>]



[Husqvarna Automower]



雪の取り込み風景 (除雪ロボット前部)

# Introduction

- One agent vs multiple agents
  - One combine harvester is fine
  - Two are faster



[[www.deere.com](http://www.deere.com)]

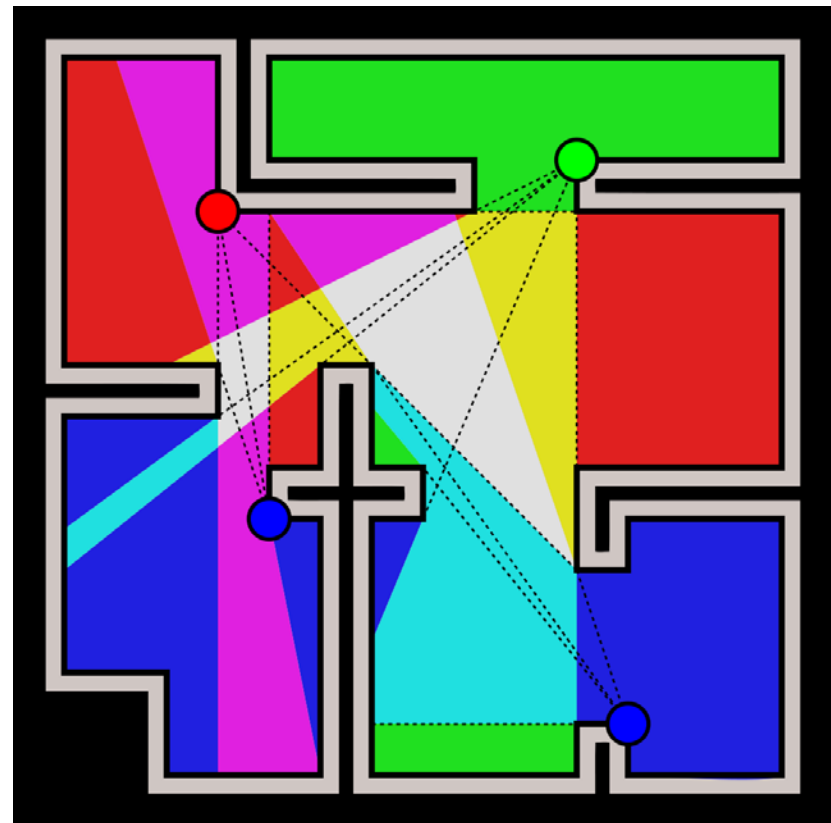
- But many are even better

# Introduction

## ■ Static coverage

- A set of agents is deployed to cover an area
- Static problem, also known as deployment
- Examples:
  - Surveillance / Monitoring:
    - ❖ Art gallery problem

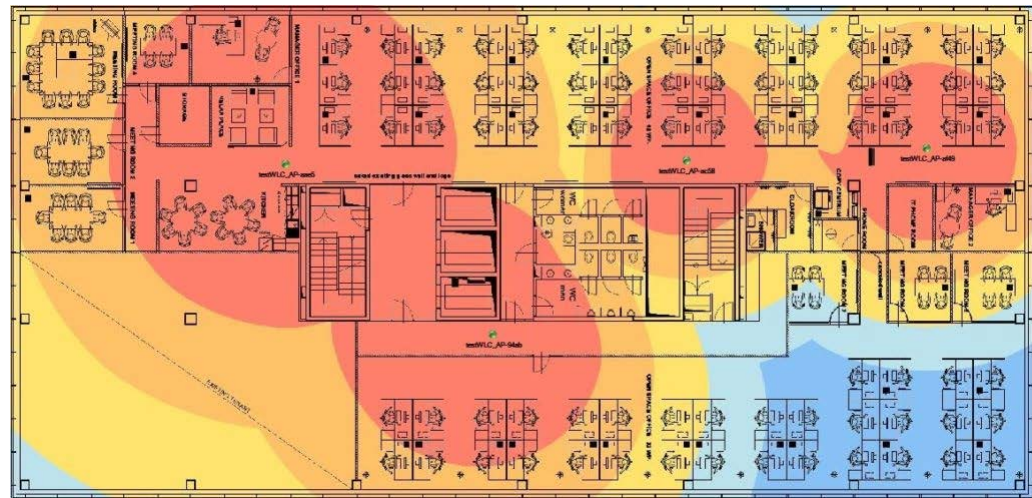
Placing four guards at the given points will guard the entire museum.



[[https://en.wikipedia.org/wiki/Art\\_gallery\\_problem](https://en.wikipedia.org/wiki/Art_gallery_problem)]

# Introduction

- Static coverage
  - A set of agents is deployed to cover an area
  - Static problem, also known as deployment
  - Examples:
    - WiFi coverage

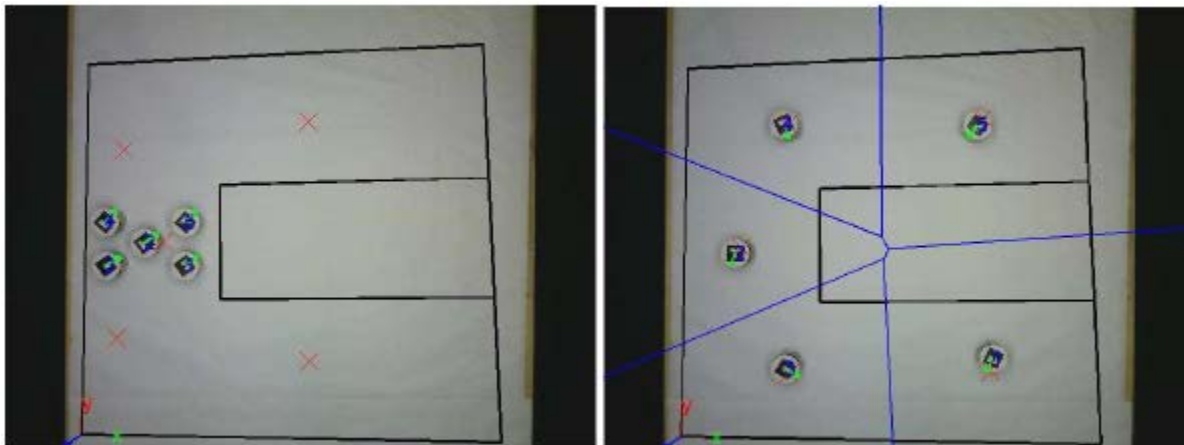


[www.ittsystems.com]

# Introduction

## ■ Static coverage (Bibliography)

- ❑ Breitenmoser, A., Schwager, M., Metzger, J.-C., Siegwart, R., Rus, D., Voronoi coverage of non-convex environments with a group of networked robots. ICRA, 2010
- ❑ Gusrialdi, A., Hirche, S., Hatanaka, T., Fujita, M., Voronoi based coverage control with anisotropic sensors. ACC 2008
- ❑ Popa, D. O., Helm, C., Stephanou, H. E., Sanderson, A. C., Robotic deployment of sensor networks using potential fields. ICRA, 2004
- ❑ Cassandras, C. G., Li, W., Sensor networks and cooperative control. European Journal of Control 11 (4-5), 436-463, 2005



# Introduction

- Dynamic coverage
  - If the number of agents is not enough to cover the area in one go, a dynamic approach is required
  - Path planning
  - Multiple agents coordination



[Roomba robot]



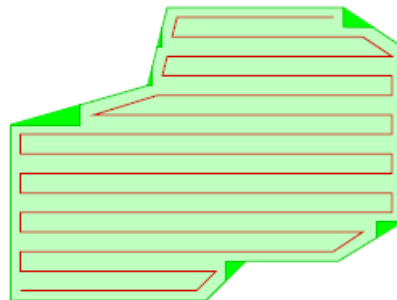
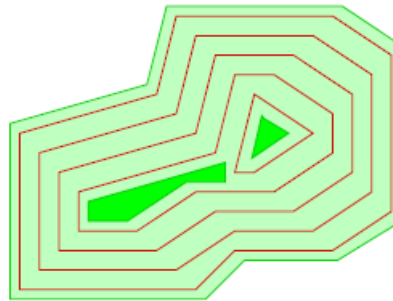
[<https://dronelife.com>]



# Introduction

## ■ Dynamic coverage (Bibliography)

- Arkin, E. M., Fekete, S. P., Mitchell, J. S. B., Approximation algorithms for lawn mowing and milling. Computational Geometry: Theory and Applications 17 (1-2), 2000, 25-50.
- Choset, H., 2001. Coverage for robotics - a survey of recent results. Annals of Mathematic and Artificial Intelligence 31 (1-4), 113-126.



# Introduction

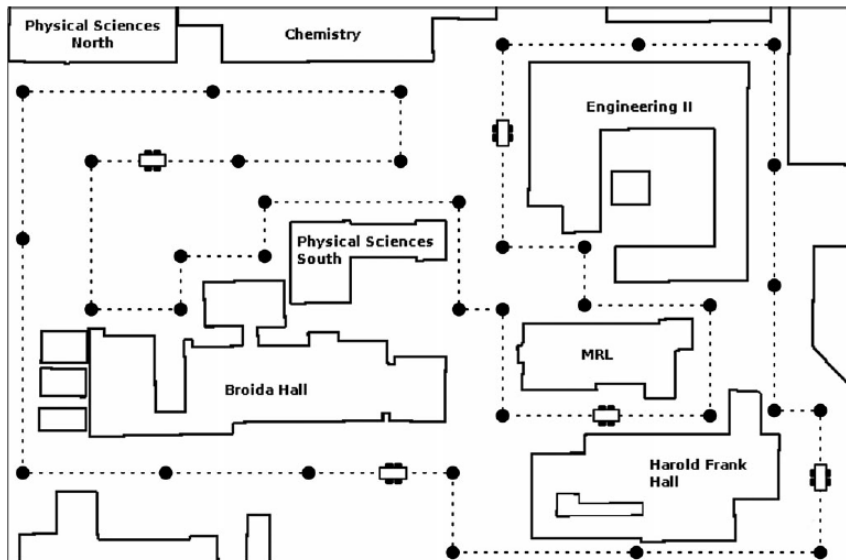
## ■ Persistent coverage

- The achieved coverage degrades (or decays) with time
  - Lawn mowing: The lawn grows
  - Snow removal: The snow continues falling
  - Surveillance: Thieves can appear at any time
  - Cleaning: The room gets dirty because there are people around
- To keep some level of coverage the agents' actions have to be maintained in time

# Introduction

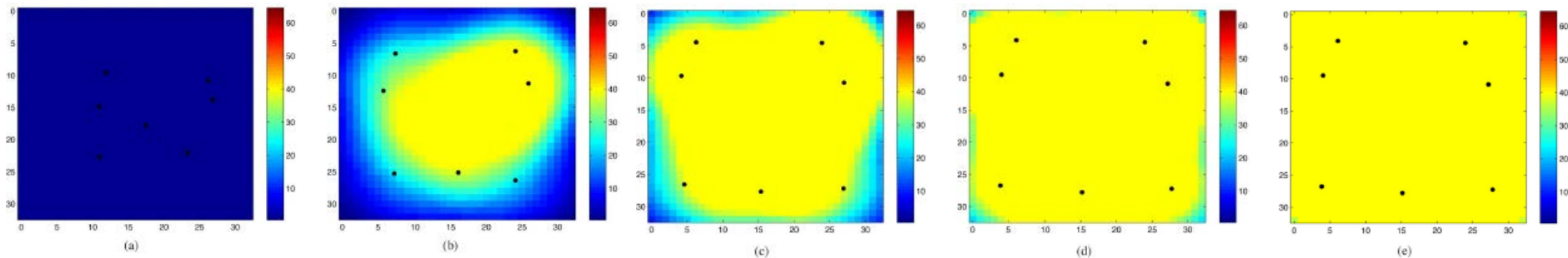
## ■ Distributed persistent coverage (Bibliography)

- F. Pasqualetti, J. W. Durham, and F. Bullo. Cooperative patrolling via weighted tours: Performance analysis and distributed algorithms. *IEEE Trans. on Robotics*, 28(5):1181–1188, 2012
- D. Portugal and R. P. Rocha. Distributed multi-robot patrol: A scalable and fault-tolerant framework. *Robotics and Autonomous Systems*, 61(12):1572–1587, 2013



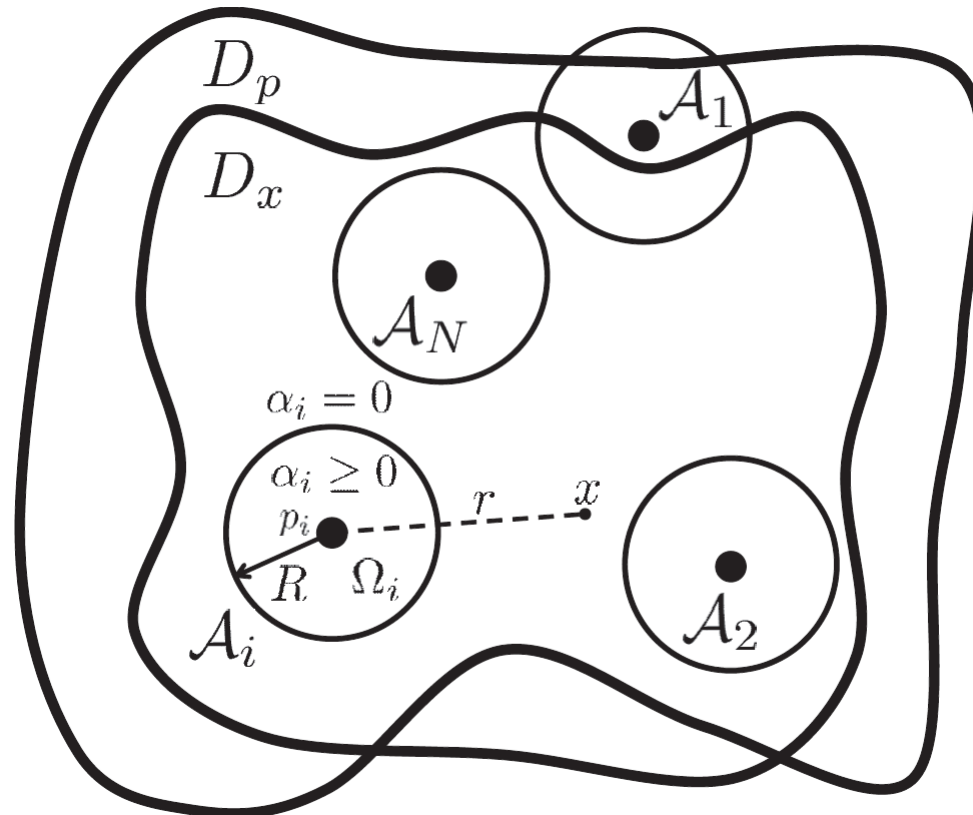
# Introduction

- Distributed persistent coverage (Bibliography)
  - Hokayem, P., Stipanovic, D., Spong, M., On persistent coverage control. IEEE Conference on Decision and Control, 2007
  - I. I. Hussein and D. M. Stipanovic, “Effective coverage control for mobile sensor networks with guaranteed collision avoidance,” IEEE Trans. on Control Systems Technology, vol. 15, no. 4, pp. 642-657, 2007



# Problem formulation

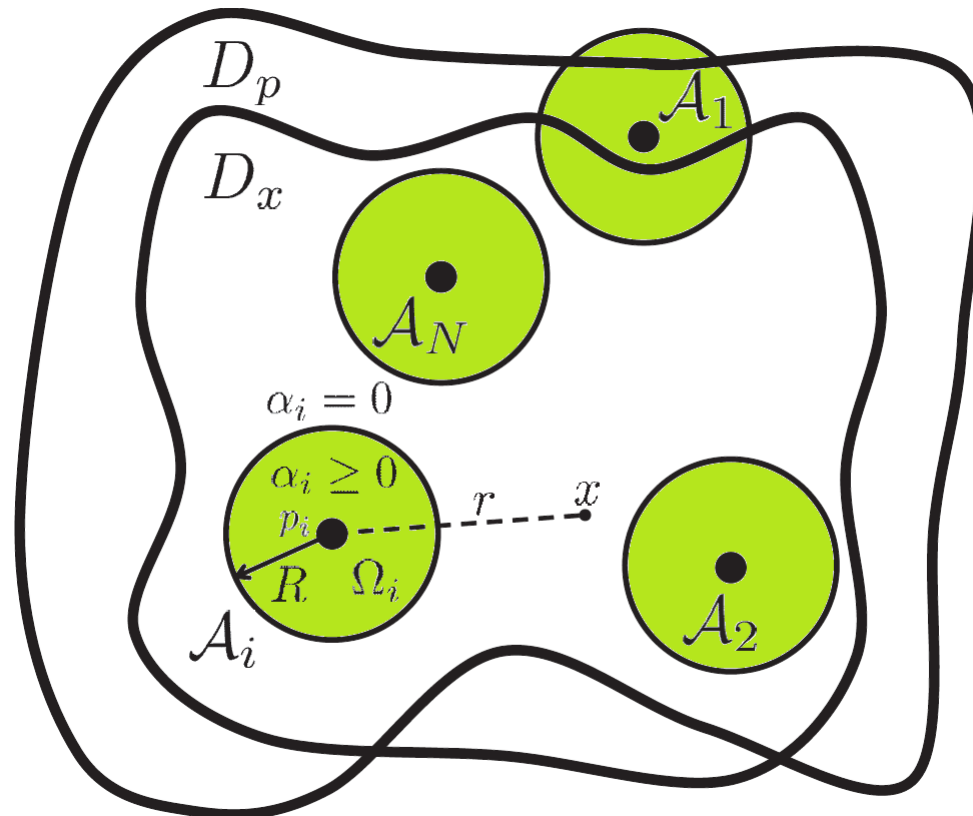
- Problem formulation of the persistent coverage task



# Problem formulation

## ■ The agents:

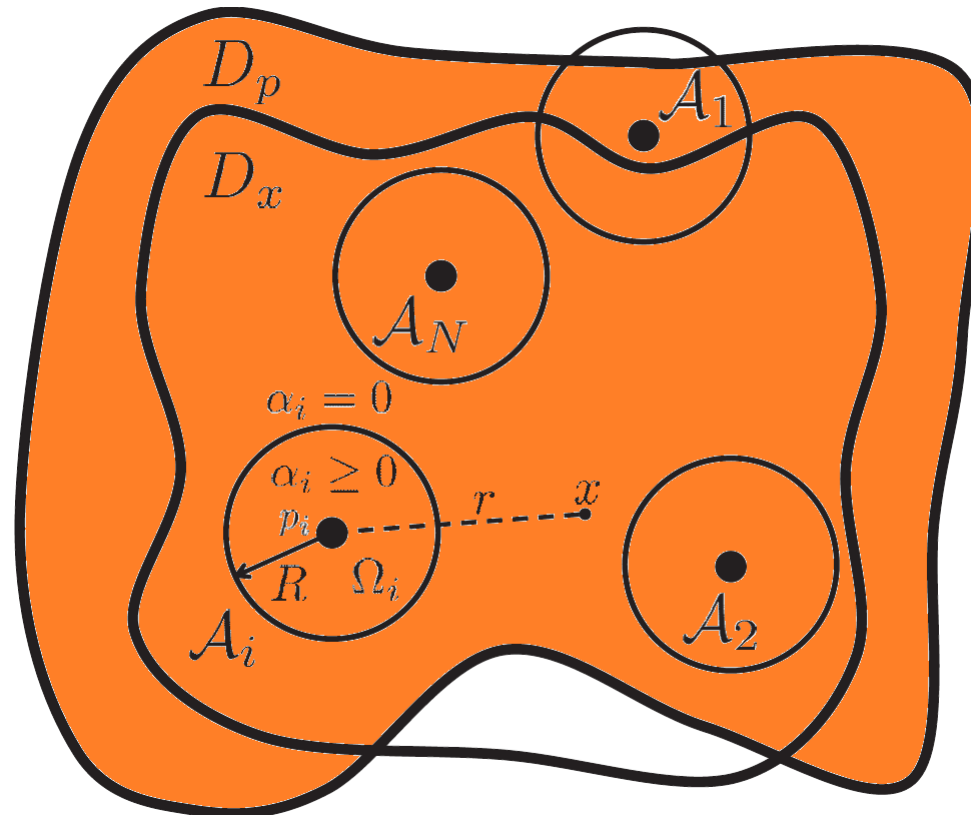
- Let us consider a team of  $N$  agents  $A = \{A_1, \dots, A_N\}$
- The coverage action of each agent is performed in domain  $\Omega_i$



# Problem formulation

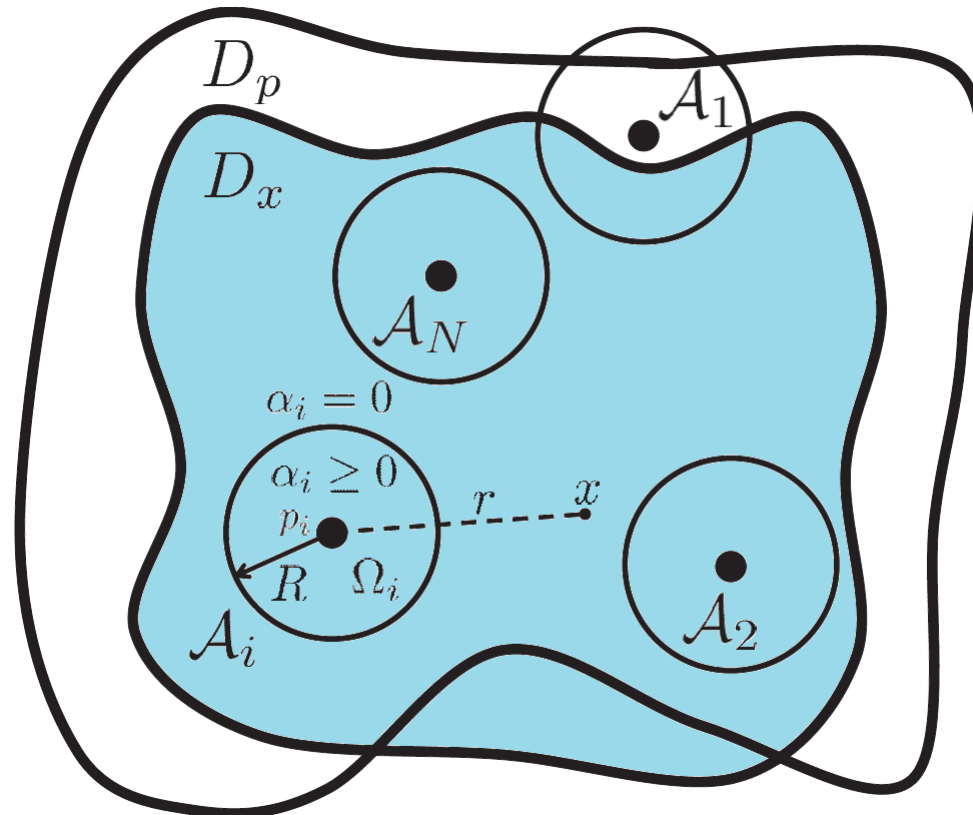
## ■ The agents:

- The agents move in a domain  $D_p$
- Position of the agent  $i$ :  $p_i(t) = [p_{ix}, p_{iy}]^T$
- Mobile agents are holonomic:  $\dot{p}_i = u_i$  with  $u_i$  the input motion



# Problem formulation

- The environment:
  - The goal is to reach a desired coverage level for all points in domain  $D_x$





# Problem formulation

## ■ The environment:

### □ Evolution of the coverage level

$$\frac{\partial \Lambda}{\partial t} = A \cdot \Lambda + B \cdot \alpha$$

$\Lambda(x, t) \in \mathbb{R}^+$ : Coverage Level

$\alpha(x, t) \in \mathbb{R}^+$ : Coverage action

$A \in \mathbb{R}$ : state gain  $A < 0$

$B \in \mathbb{R}$ : input gain  $B > 0$

### □ Error coverage (Domain $D_x$ )

$$e_{D_x} = \int_{D_x} \Phi \cdot (\Lambda^* - \Lambda)^2 dx$$

$D_x \subset \mathbb{R}^2$ : Coverage Domain

$\Lambda^*(x) \in \mathbb{R}^+$ : Coverage Objective

$\Phi(x) \in (0, 1]$ : Coverage Priority

### □ Coverage priority $\Phi \in (0, 1]$ is the priority to cover each point

- Example: watering the garden

# Problem formulation

## ■ The goal:

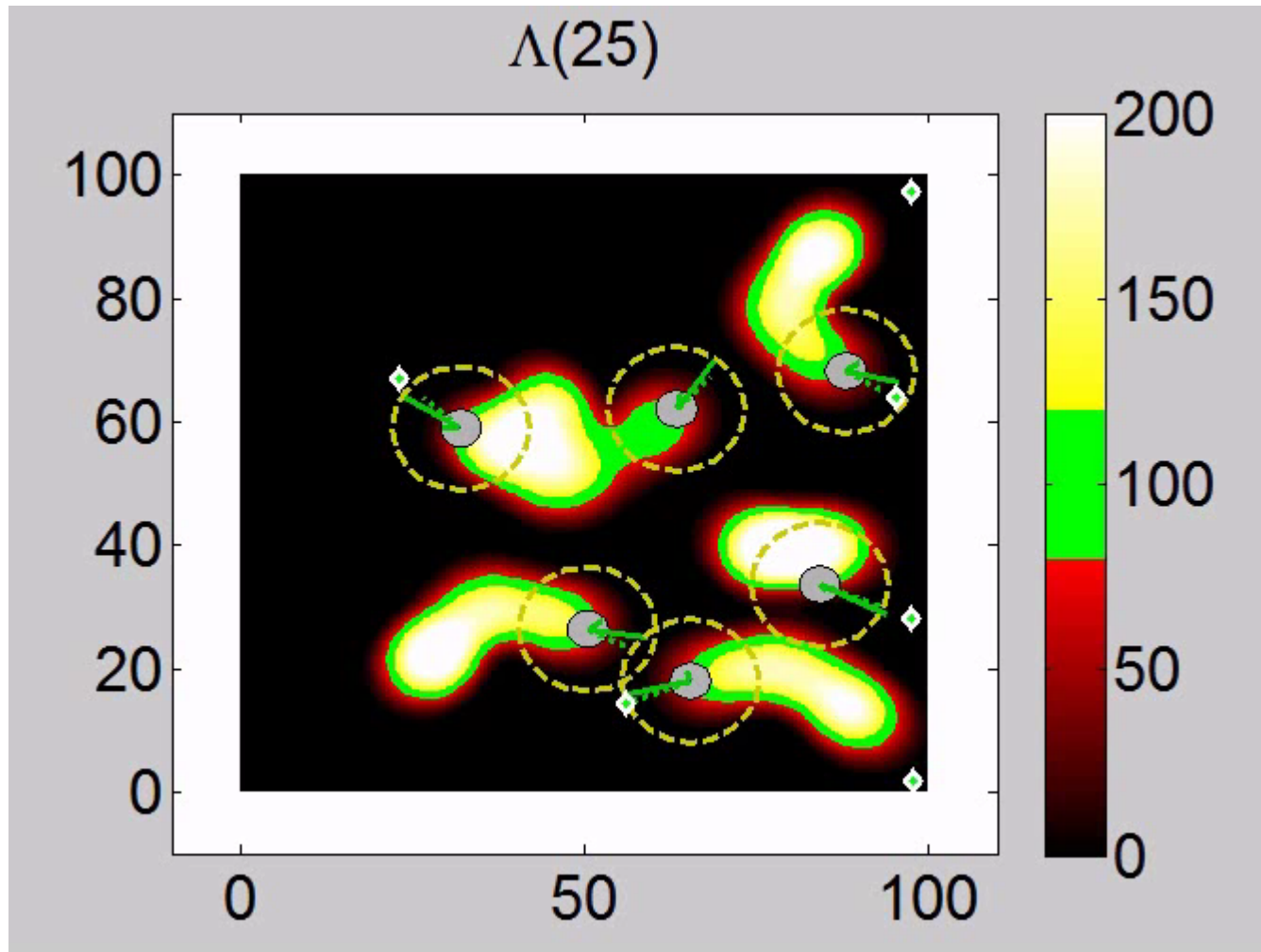
- The aim of our problem is to minimize the coverage error of the domain by reaching the desired coverage level  $\Lambda^*(x)$  all over the domain
  - No more, no less:
    - ❖ There are applications that require a particular coverage level, and higher coverage leads to a waste of energy, as for example cleaning, or to bad results as painting.



[Y. Tang, W. Chen 2015]

# Problem formulation

- The goal:

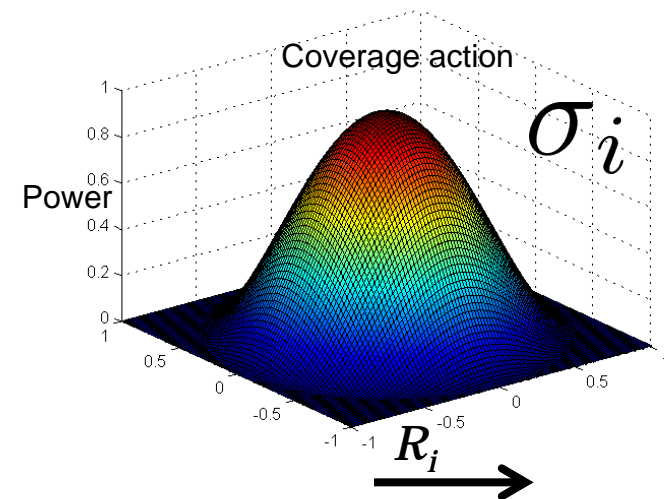
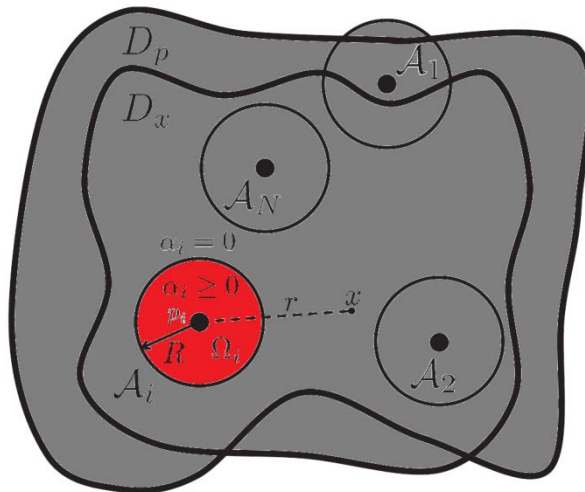


# Coverage action control

## ■ Coverage action

- Developed in domain  $\Omega_i$  of each agent
- We consider circular actuators with range  $R_i$
- Coverage function:

$$\begin{cases} \alpha_i \geq 0 & \text{if } r < R_i \quad (x \in \Omega_i) \\ \alpha_i = 0 & \text{if } r \geq R_i \quad (x \notin \Omega_i) \end{cases}$$



$\sigma_i$  = Shape of the coverage action

$\int_{\Omega_i} \sigma_i dx = 1$ : Normalization of the shape of the coverage action

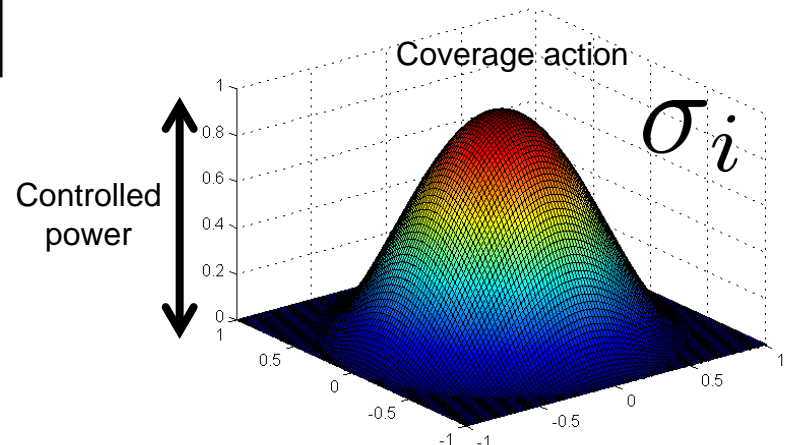
# Coverage action control

- Variable coverage power control
  - Adaptive and efficient action
- Tries to reduce the error over the actuator area with an action  $\alpha_i$  proportional  $C$  to the weighted error  $\sigma_i \cdot (\Lambda^* - \Lambda)$  of the coverage domain of each agent  $\Omega_i$

$$\alpha_i = K \cdot \sigma_i$$

- Weighted error in the agent domain

$$K = C \cdot \left[ \int_{\Omega_i} B \cdot \Phi \cdot \sigma_i \cdot (\Lambda^* - \Lambda) dx \right]^{2 \cdot q - 1}$$

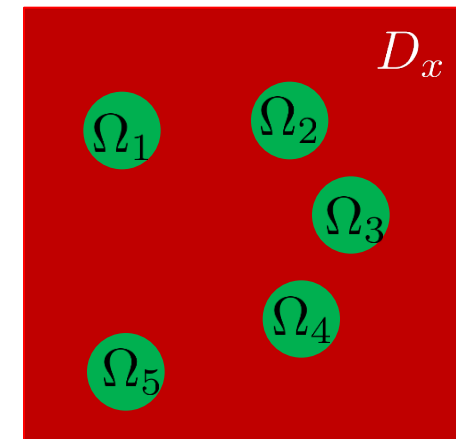


# Coverage action control

- Variable coverage power control
  - Evolution of the error with time:

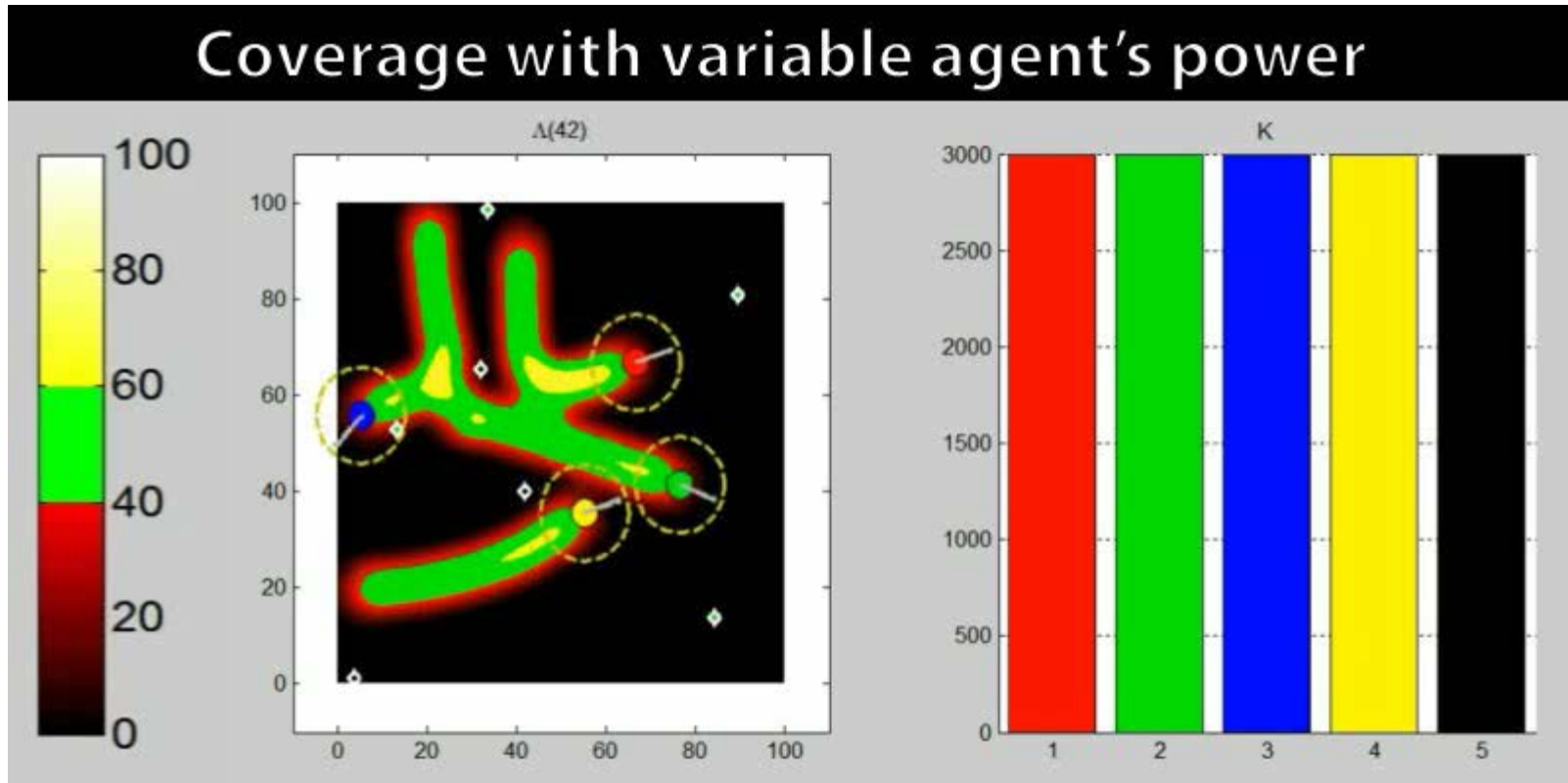
$$\begin{aligned}
 \frac{de_{D_x}}{dt} &= -2 \cdot \left[ \int_{D_x} A \cdot \Phi \cdot \Lambda \cdot (\Lambda^* - \Lambda) dx + \int_{D_x} B \cdot \alpha_i \cdot \Phi \cdot (\Lambda^* - \Lambda) dx \right] \\
 &= -2 \cdot \left[ \int_{D_x} A \cdot \Phi \cdot \Lambda \cdot (\Lambda^* - \Lambda) dx + K \cdot \left( \int_{\Omega_i} B \cdot \sigma_i \cdot \Phi \cdot (\Lambda^* - \Lambda) dx \right) \right] \\
 &= -2 \cdot \left[ \int_{D_x} A \cdot \Phi \cdot \Lambda \cdot (\Lambda^* - \Lambda) dx + C \cdot \left( \int_{\Omega_i} B \cdot \sigma_i \cdot \Phi \cdot (\Lambda^* - \Lambda) dx \right)^{2 \cdot q} \right]
 \end{aligned}$$

- The coverage decays / vanishes and makes the error grow throughout the domain
- The design of the coverage action guarantees that in the actuator domain of agents error decreases



# Coverage action control

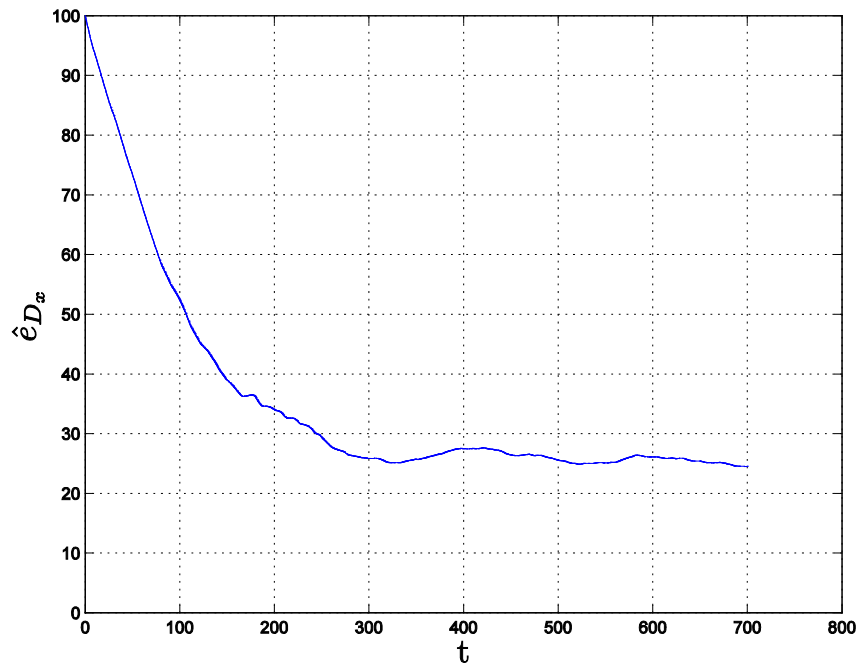
- Variable coverage power control



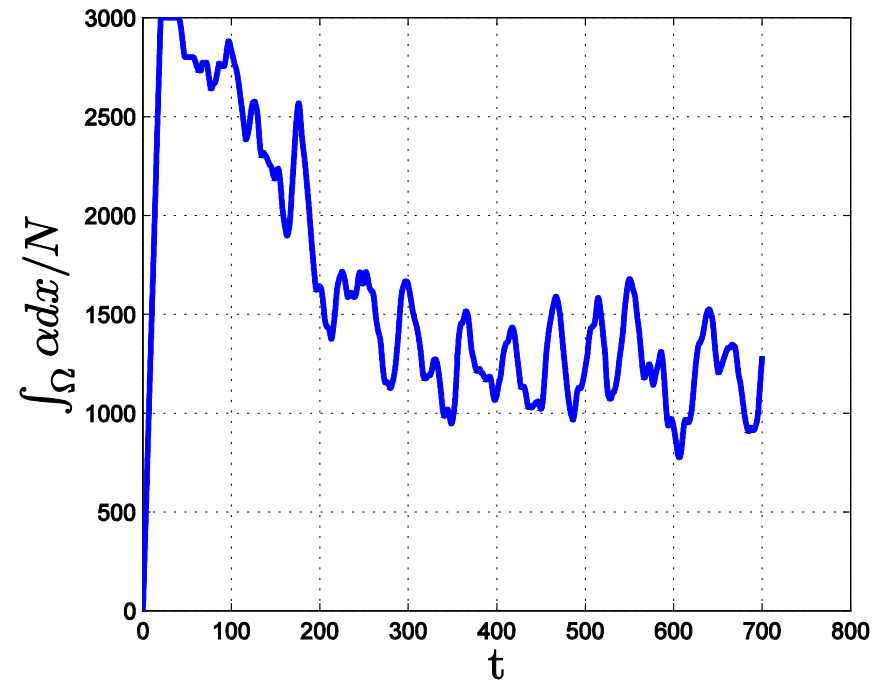
# Coverage action control

## ■ Variable coverage power control

Error of the domain



Average coverage power

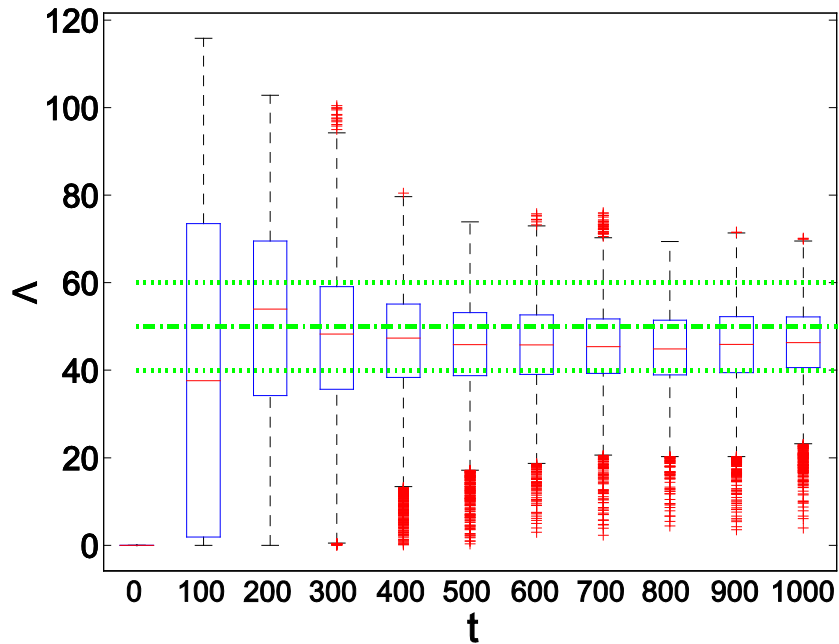




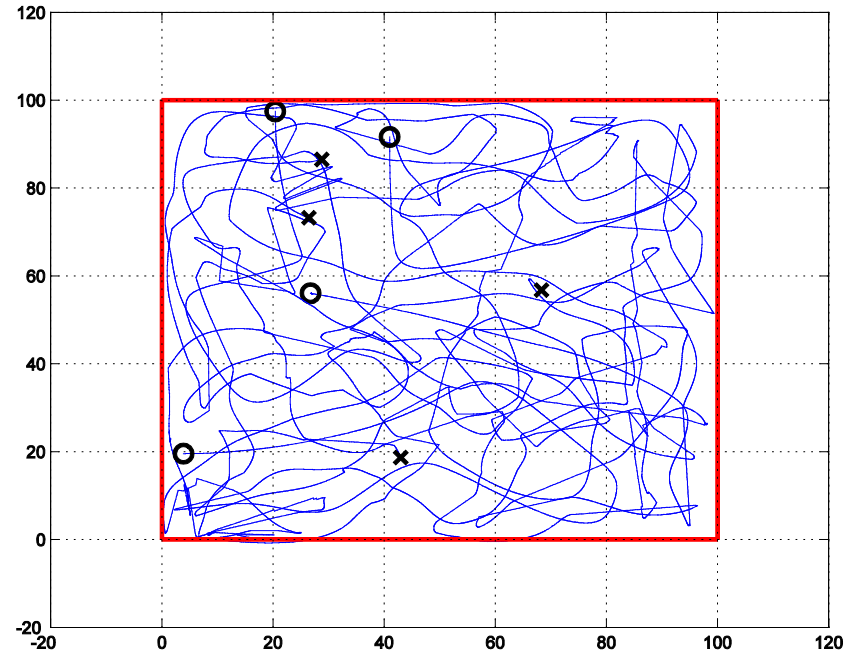
# Coverage action control

## ■ Variable coverage power control

Boxplot along time of the coverage level



Agent's paths



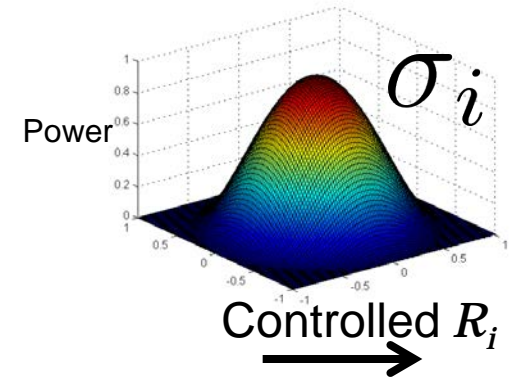
# Coverage action control

- Variable coverage range control

- Range of the actuator:  $R_i$

- Taking the second derivative over time of the error

$$\frac{\partial^2 e_{D_x}}{\partial t^2} = -2 \left[ \int_{D_x} A \cdot \Phi \cdot \frac{\partial \Lambda}{\partial t} \cdot (\Lambda^* - 2 \cdot \Lambda) dx + 2 \cdot q \cdot C \cdot \left( \int_{\Omega_i} B \cdot \Phi \cdot \sigma_i \cdot (\Lambda^* - \Lambda) dx \right)^{2 \cdot q - 1} \cdot \int_{\Omega_i} B \cdot \Phi \cdot \left( \frac{\partial \sigma_i}{\partial R} \cdot \frac{\partial R}{\partial t} \cdot (\Lambda^* - \Lambda) - \sigma_i \cdot \frac{\partial \Lambda}{\partial t} \right) dx \right]$$



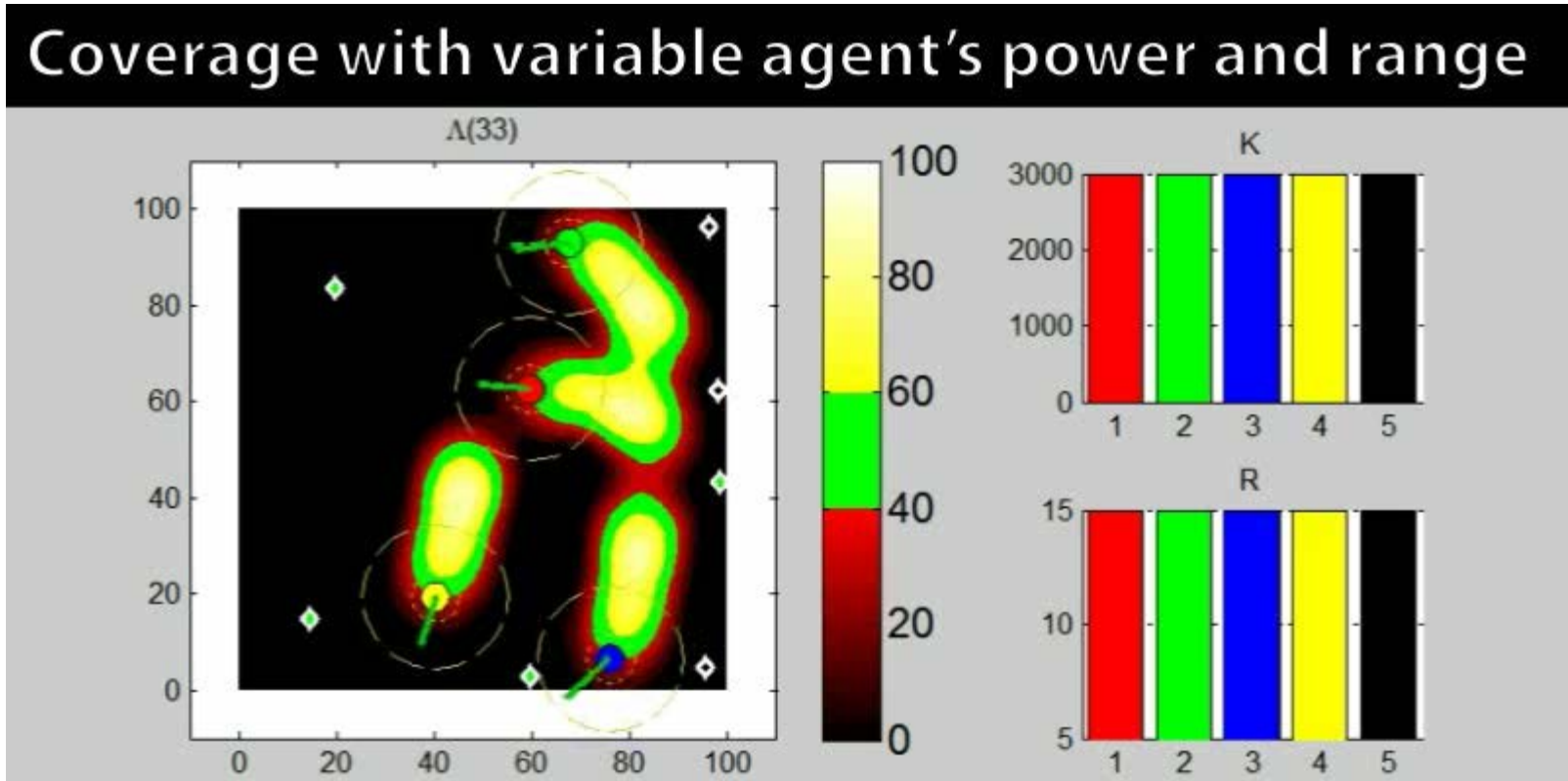
- Design a range control that makes the second of the addend always negative

$$\frac{\partial R}{\partial t} = k_i^R \int_{\Omega_i} B \cdot \Phi \cdot \sigma_i \cdot (\Lambda^* - \Lambda) dx \cdot \int_{\Omega_i} B \cdot \Phi \cdot \frac{\partial \sigma_i}{\partial R} \cdot (\Lambda^* - \Lambda) dx,$$

- The idea is to make the second derivative of the error as low as possible to reduce the coverage error

# Coverage action control

- Variable coverage range control

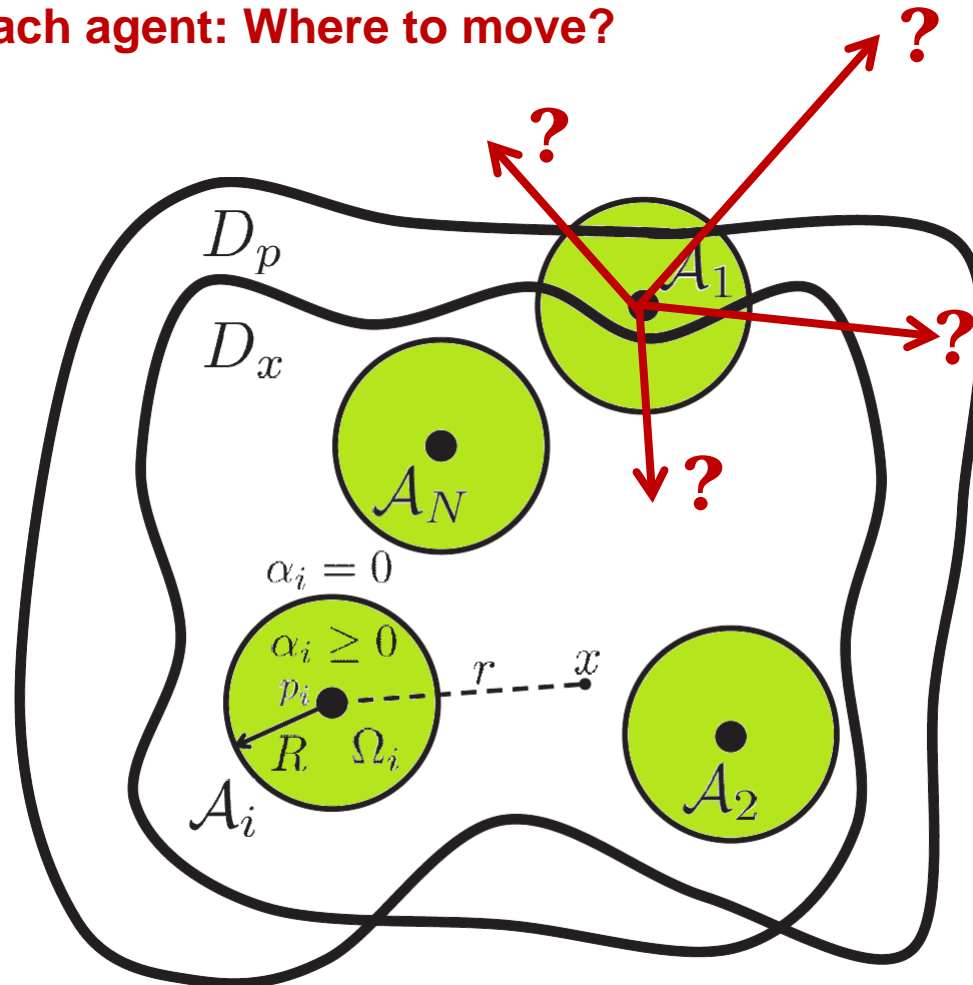


# Motion control

## ■ Motion action control

□ The objective of the motion control law is to keep decreasing the error

➤ **For each agent: Where to move?**



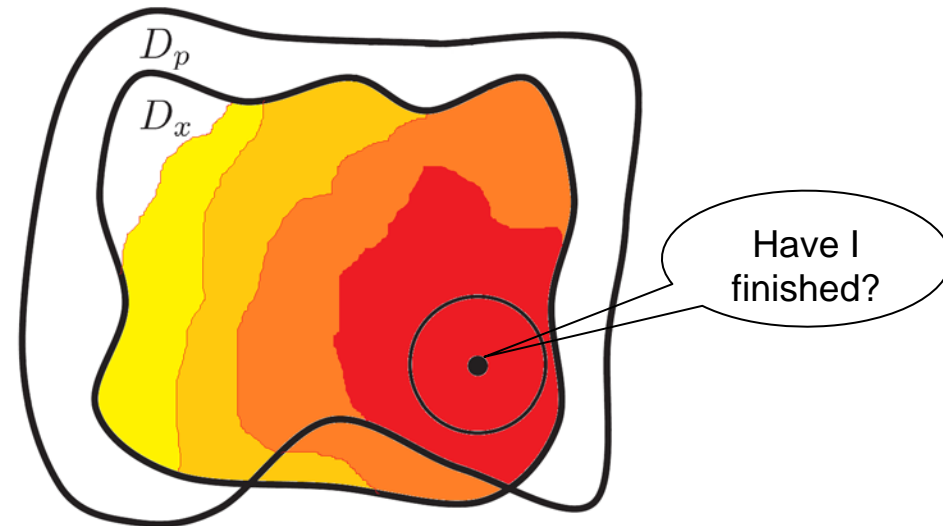
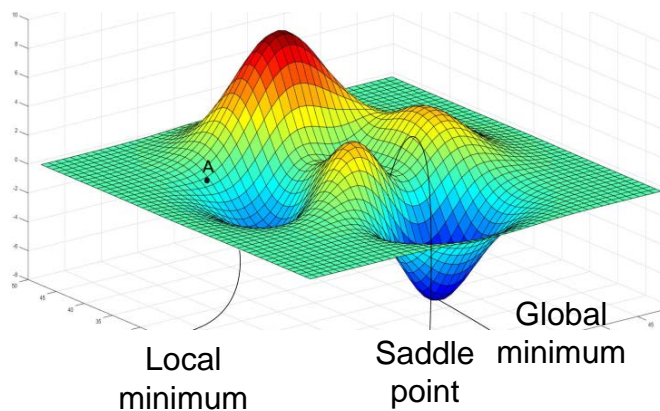
# Motion control

## ■ Motion action control

- Local control law based on gradient of the error with respect to agent position:

$$u_i^{loc}(t) = \frac{\partial}{\partial p_i} \left( \frac{\partial e_{D_x}}{\partial t} \right) = -4 \cdot q \cdot C \cdot \left( \int_{\Omega_i} B \cdot \sigma_i \cdot \Phi \cdot (\Lambda^* - \Lambda) dx \right)^{2 \cdot q - 1} \cdot \int_{\Omega_i} B \cdot \Phi \cdot \frac{\partial \sigma_i}{\partial r} \cdot \frac{p_i - x}{\|p_i - x\|} \cdot (\Lambda^* - \Lambda) dx$$

- However, a local control law is known to get stuck in local minima



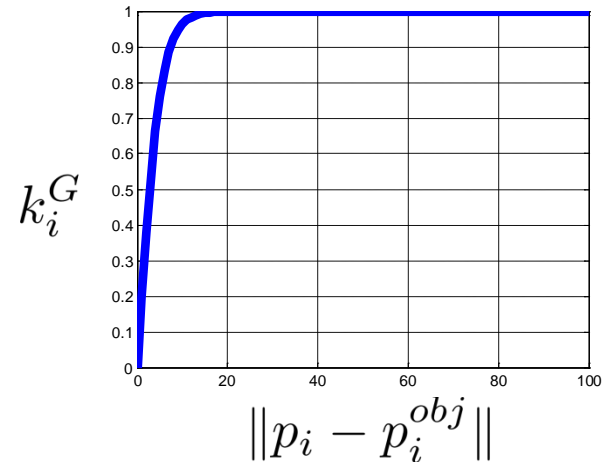
# Motion control

## ■ Motion action control

- Global control law to avoid local minima tries to reach global targets

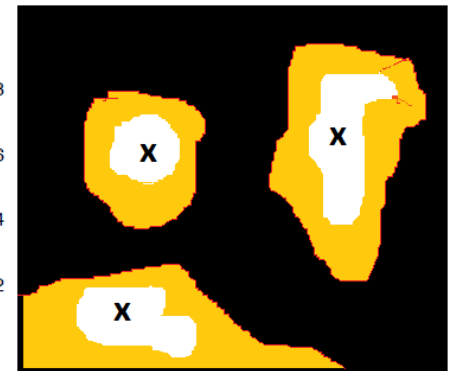
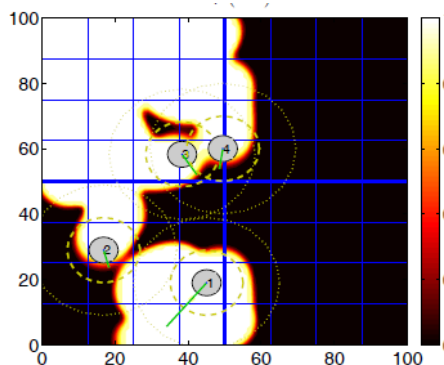
$$u_i^{glo} = k_i^G \cdot \frac{p_i - p_i^{obj}}{\|p_i - p_i^{obj}\|}$$

$$k_i^G = \tanh\left(\frac{2 \cdot d_i^{obj}}{R}\right)$$



- Selection of global objectives
  - ❖ Hierarchical grid of the domain
  - ❖ Blob analysis of the domain

$$\underline{p_i^{obj}}$$



# Motion control

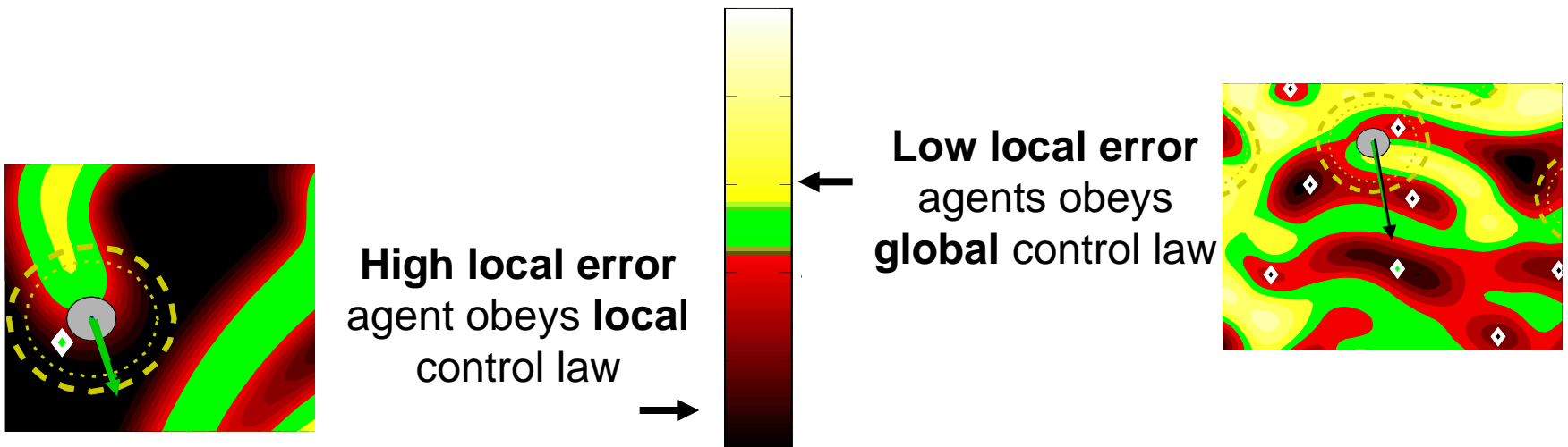
## ■ Motion action control

- Combination of local control law and global control law
- Based on the coverage error of the actuator domain  $\Omega_i$  we introduce a normalized local error

$$\varsigma_{\Omega_i} = \int_{\Omega_i} \Phi \cdot \sigma_i \cdot \frac{(\Lambda^* - \Lambda)}{\Lambda^*} dx \quad \varsigma_{\Omega_i}(t) \in (-\infty, 1]$$

- Indicates that agent's neighborhood is satisfactorily covered when it is negative or 0

$$u_i^{cov} = (\varsigma_{\Omega_i}^+)^{\beta} \cdot \hat{u}_i^{loc} + (1 - (\varsigma_{\Omega_i}^+)^{\beta}) \cdot u_i^{glo} \quad \varsigma_{\Omega_i}^+ = \max(0, \varsigma_{\Omega_i})$$



# Motion control

## ■ Motion action control

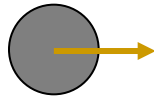
- Finally, the velocity control law

$$u_i = k_i \cdot (1 - \varsigma_{\Omega_i}^+) \cdot u_i^{cov}$$

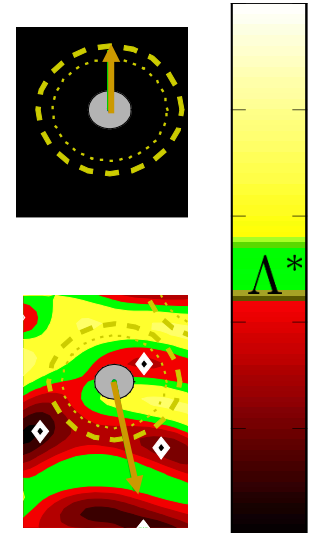
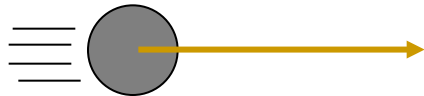
- Bounded by the motion gain  $k_i$

- Based on the coverage error of the actuator domain  $\Omega_i$

- High error: slow down and develop coverage carefully



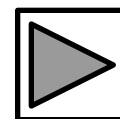
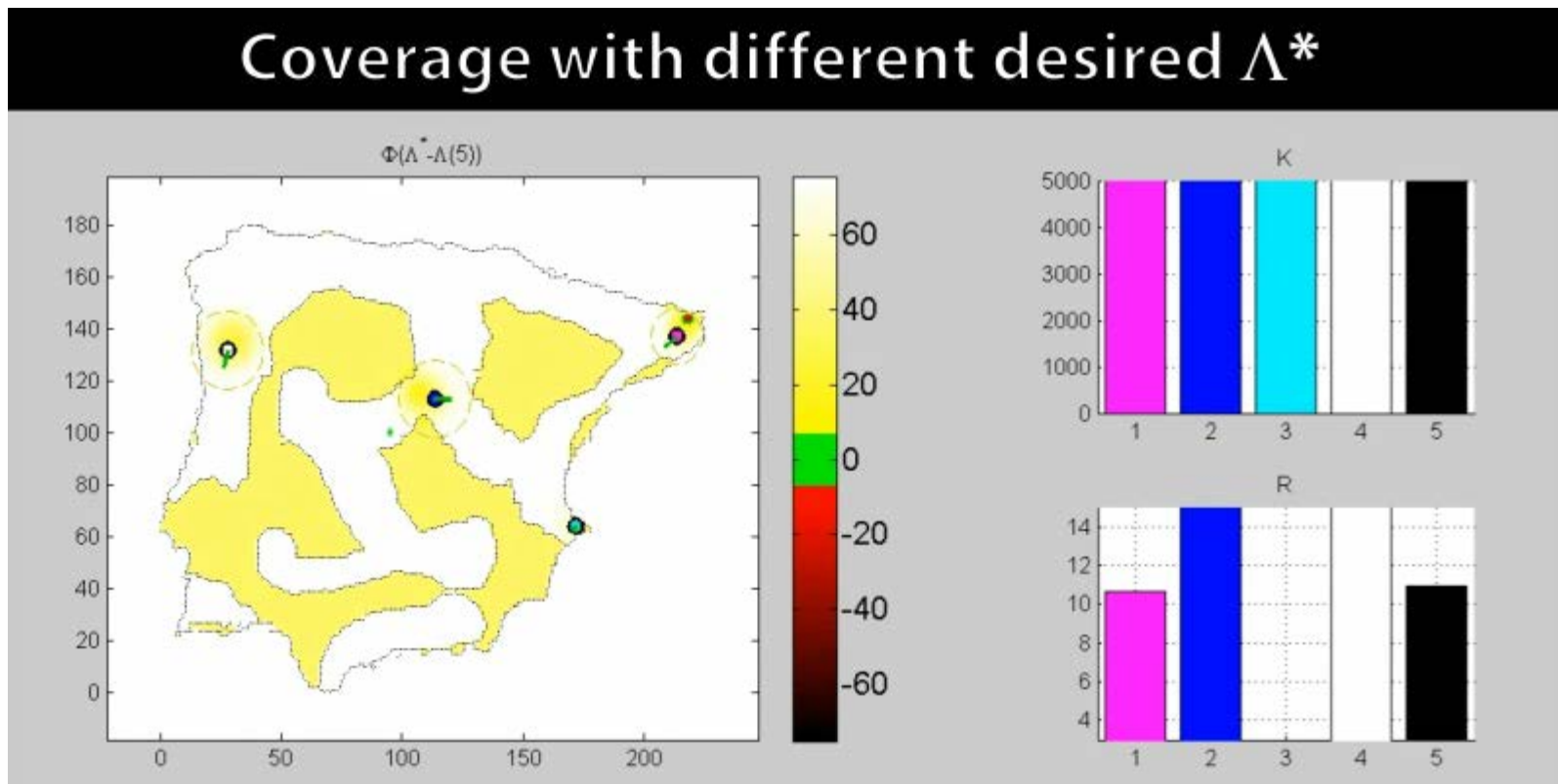
- Low error: speed up to reach zones with higher error





# Motion control

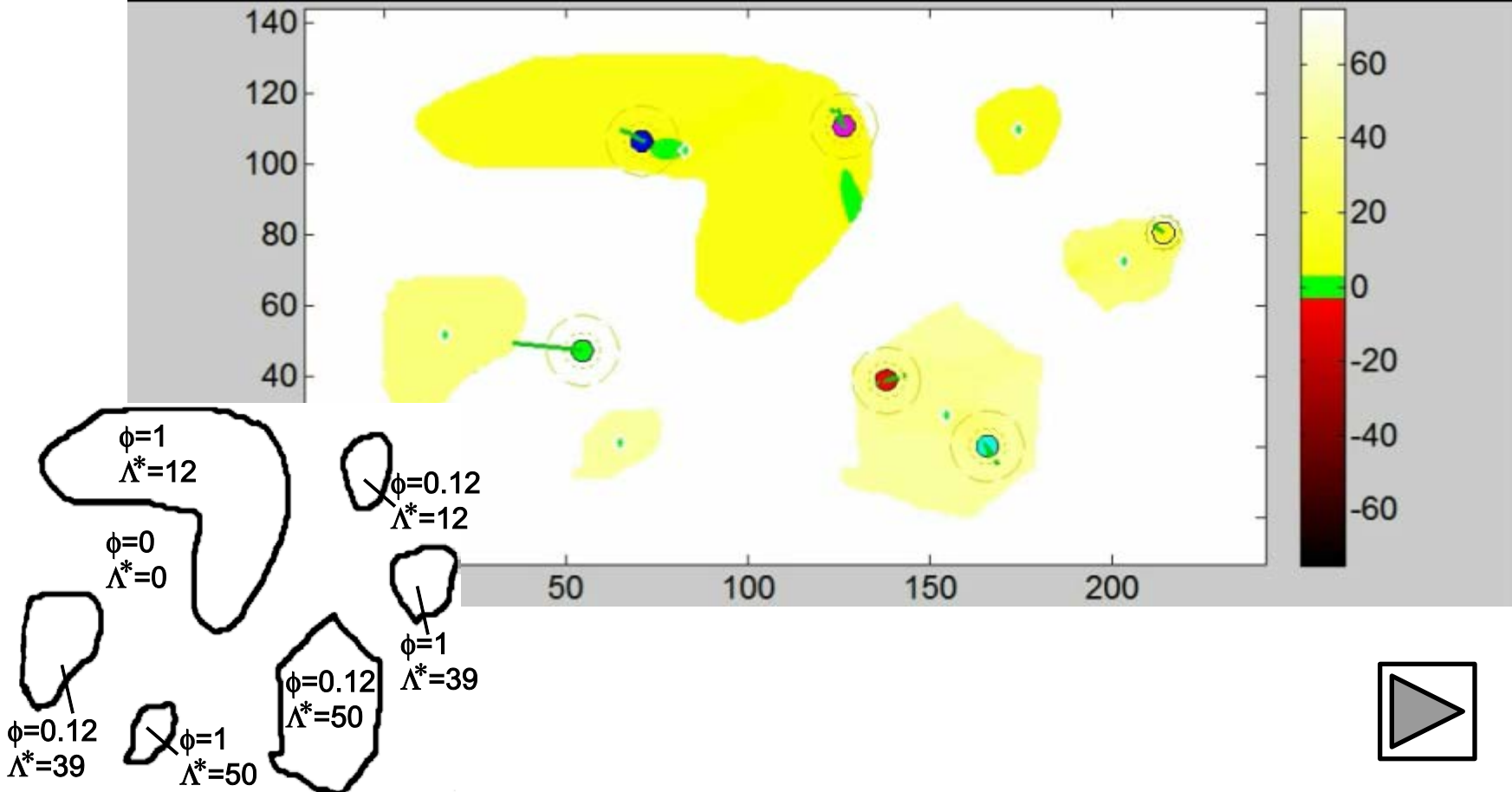
## ■ Example



# Motion control

## ■ Example

### Coverage with different priorities and desired $\Lambda^*$



# Bibliography

- Some additional topics
  - Anisotropic sensors
  - Purely distributed algorithms
  - Collision avoidance

# Bibliography

- C. Franco, G. López-Nicolás, C. Sagüés, S. Llorente. Persistent coverage control with variable coverage action in multi-robot environment. IEEE Conference on Decision and Control, 2013
- C. Franco, G. López-Nicolás, C. Sagüés, S. Llorente. Adaptive action for multi-agent persistent coverage. Asian Journal of Control, vol. 18, no. 2, pp. 419-432, 2016
- C. Franco, D. M. Stipanovic, G. López-Nicolás, C. Sagüés, S. Llorente. Persistent coverage control for a team of agents with collision avoidance. European Journal of Control, vol. 22, pp. 30-45, 2015