



Multirobot Systems

Lecture

Multirobot collision avoidance

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Introduction

- Real-time collision avoidance of mobile robots
 - They should sense their environment and react independently without communication or explicit coordination, as humans do



[Darpa challenge]

Introduction

- Multirobot motion planning
 - Global planning: Trajectory from initial to final location
 - Local planning: Collision avoidance up to a short time horizon
- Dynamic motion planning for a point in the plane, with bounded velocity and arbitrary many obstacles, is intractable (NP-hard)

Introduction

- Applications:
 - Autonomous cars
 - Computer games
 - Traffic engineering
 - Crowd simulation

- Warehouses with Kiva robots to deliver stacks of products

Introduction

- Applications:
 - Autonomous cars
 - Computer games
 - Traffic engineering
 - Crowd simulation
- Shibuya pedestrian crossing (Tokyo)
 - Around a million of people per day, 47 seconds to cross the street



State of the art

■ Collision avoidance approaches:

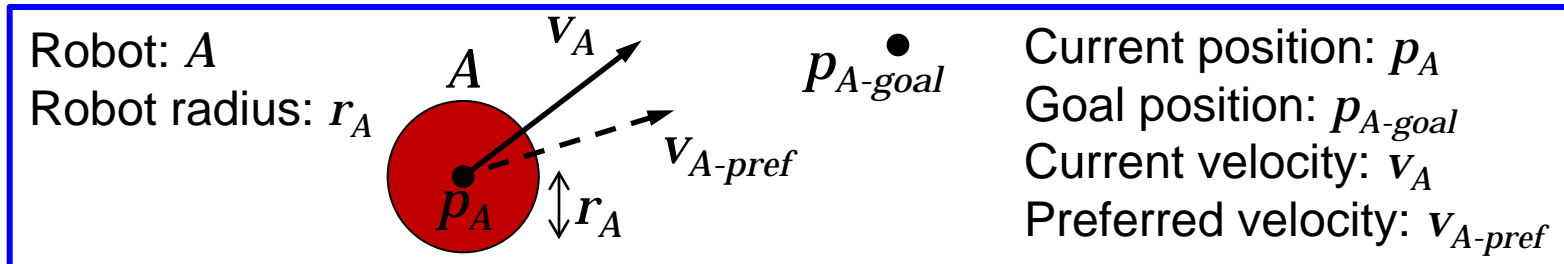
- Dynamic motion planning
- Local collision avoidance

■ Some papers:

- D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in IEEE Robotics & Automation Magazine, vol. 4, no. 1, pp. 23-33, March 1997
- L. Pallottino, V. G. Scordio, A. Bicchi and E. Frazzoli, "Decentralized cooperative policy for conflict resolution in multivehicle systems," in IEEE Transactions on Robotics, vol. 23, no. 6, pp. 1170-1183, Dec. 2007, doi: 10.1109/TRO.2007.909810.
- P. Fiorini, Z. Shiller, "Motion planning in dynamic environments using velocity obstacles," The International Journal of Robotics Research. 1998;17(7):760-772.
- J.L. Blanco, J. González and J.A. Fernández-Madrugal, "Extending obstacle avoidance methods through multiple parameter-space transformations," in Autonomous Robots, 24, 29-48 (2008). <https://doi.org/10.1007/s10514-007-9062-7>
- **J. Snape, J. P. van den Berg, S. J. Guy, D. Manocha. "The hybrid reciprocal velocity obstacle," IEEE Transactions Robotics 27(4). 696-706 (2011)**
- M. Rufli, J. Alonso-Mora and R. Siegwart, "Reciprocal collision avoidance with motion continuity constraints," in IEEE Transactions on Robotics, vol. 29, no. 4, pp. 899-912, Aug. 2013, doi: 10.1109/TRO.2013.2258733.
- L. Wang, A. D. Ames and M. Egerstedt, "Safety barrier certificates for collisions-free multirobot systems," in IEEE Transactions on Robotics, vol. 33, no. 3, pp. 661-674, Jun. 2017

Problem definition

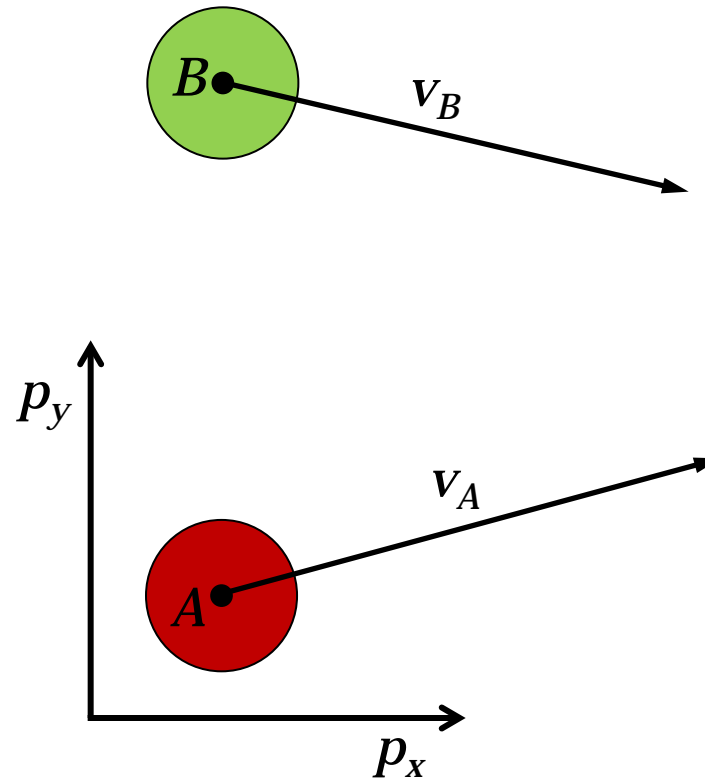
- Setup: Navigation in an environment with dynamic obstacles
 - Disc-shaped robots in the plane with fixed radius and a current velocity
 - Known to the robot and may be measured by the others
 - Dynamic and static obstacles
 - We assume they can be identified by each robot
 - Each robot has a goal location and a preferred speed
 - unknown to the other robots
 - No communications. No coordination
 - May assume the others use the same strategy for collision avoidance



- Problem:
 - Each robot computes a new velocity at each time step to move toward its goal without collisions with any other robots

Problem definition

- Given robots A and B moving in the plane, will they collide in ΔT ?



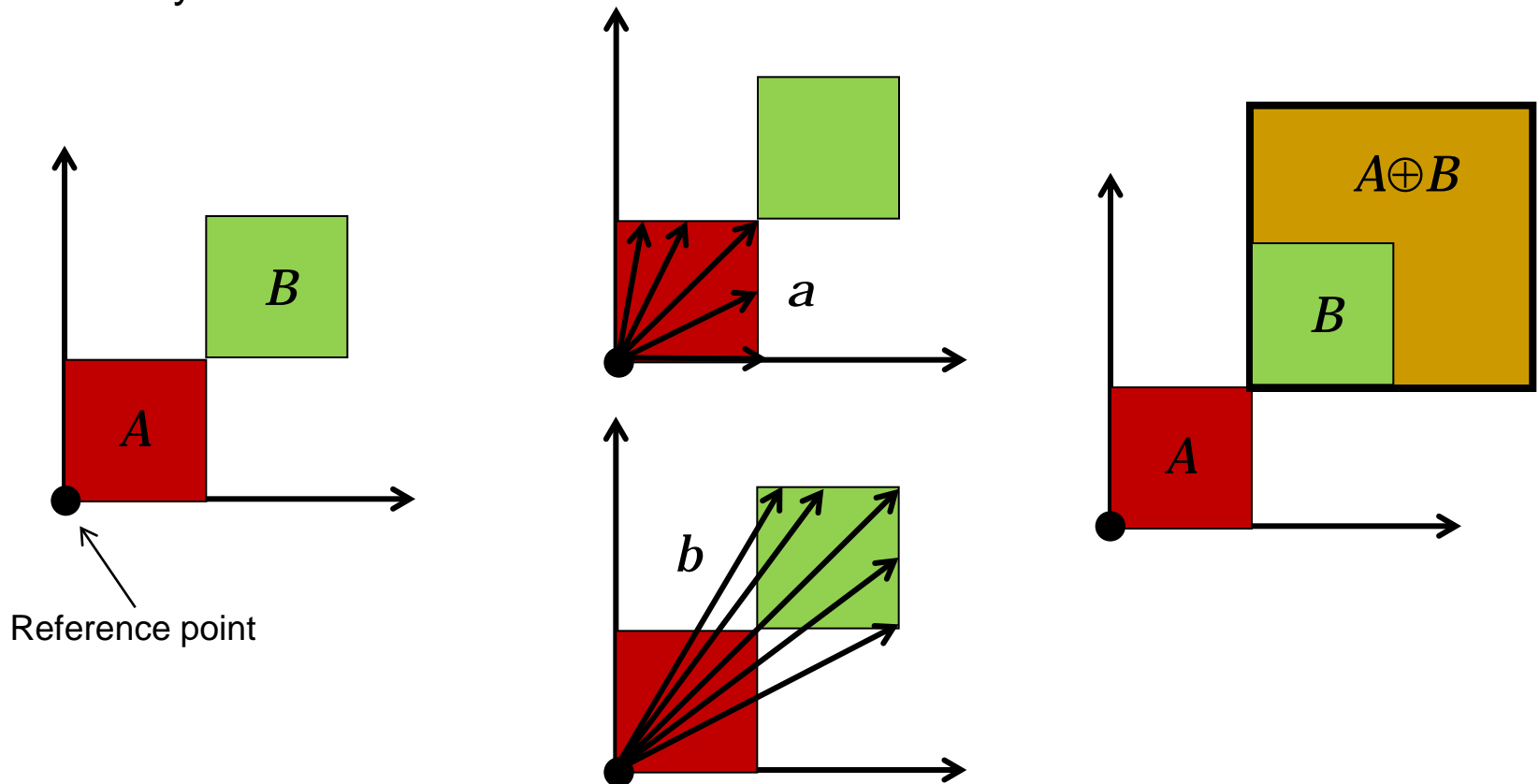
Velocity obstacle method

■ Minkowski addition

- Definition: is the sum of sets A and B in a vector space such that

$$A \oplus B = \{a + b \mid a \in A, b \in B\}$$

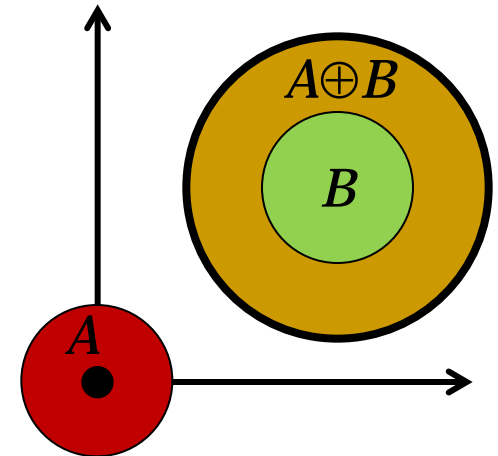
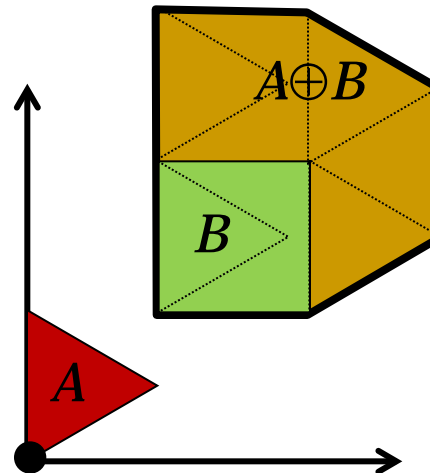
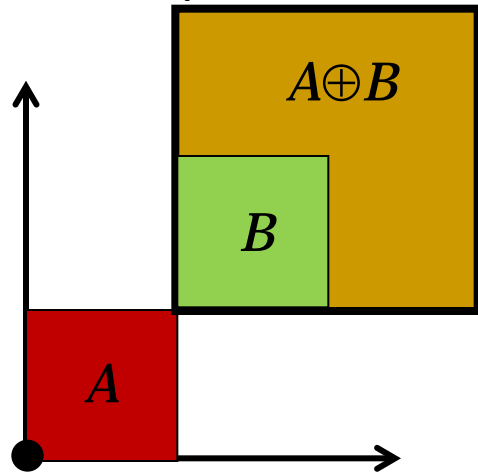
- May be referred as *dilation*



Velocity obstacle method

■ Minkowski addition

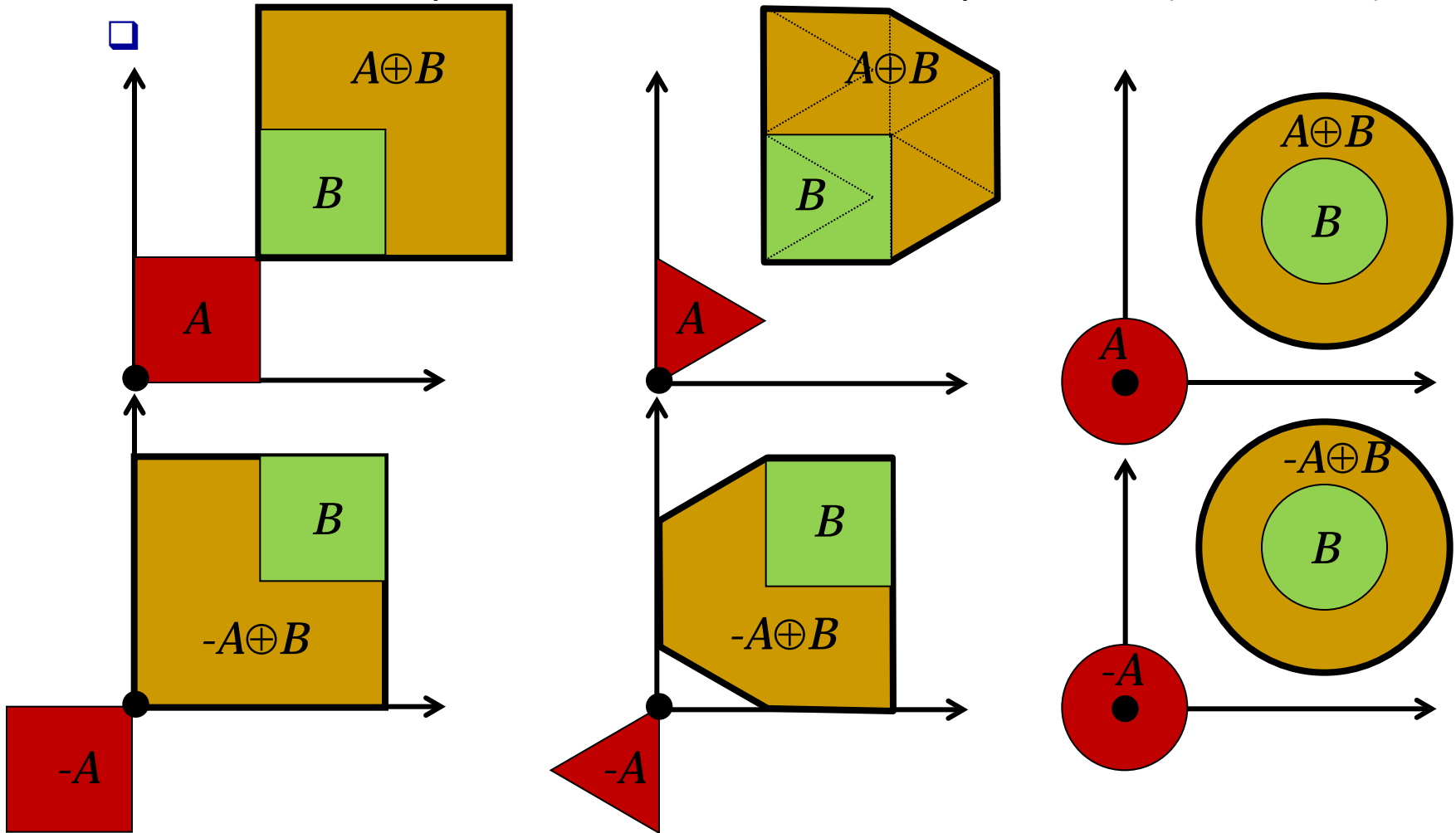
□ Examples



Velocity obstacle method

Minkowski addition

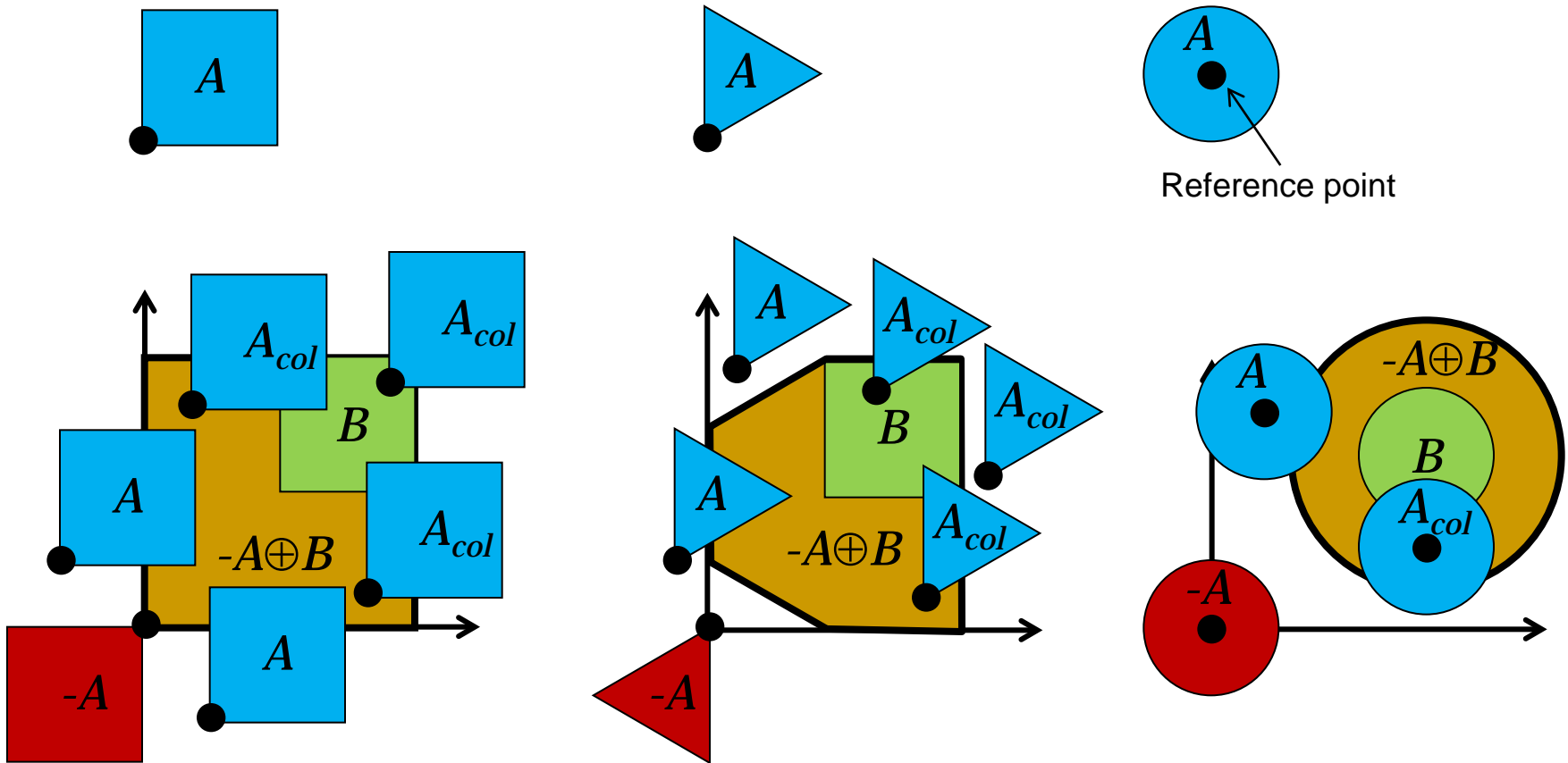
Let $-A$ denote shape A reflected in its reference point: $-A = \{ -a / a \in A \}$



Velocity obstacle method

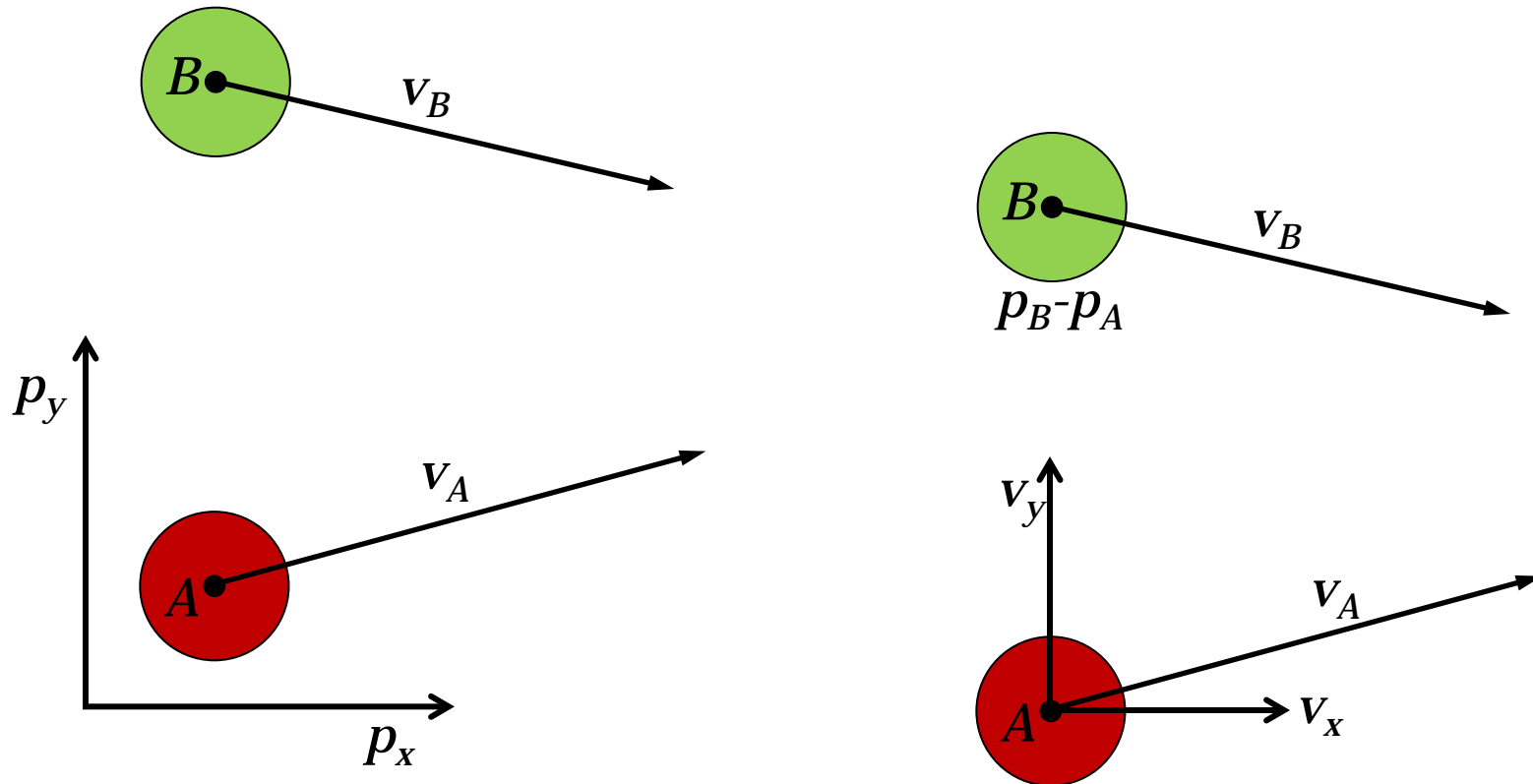
■ Minkowski addition

- Minkowski difference is related to image erosion
- There is no collision if the reference point is outside the area $-A \oplus B$



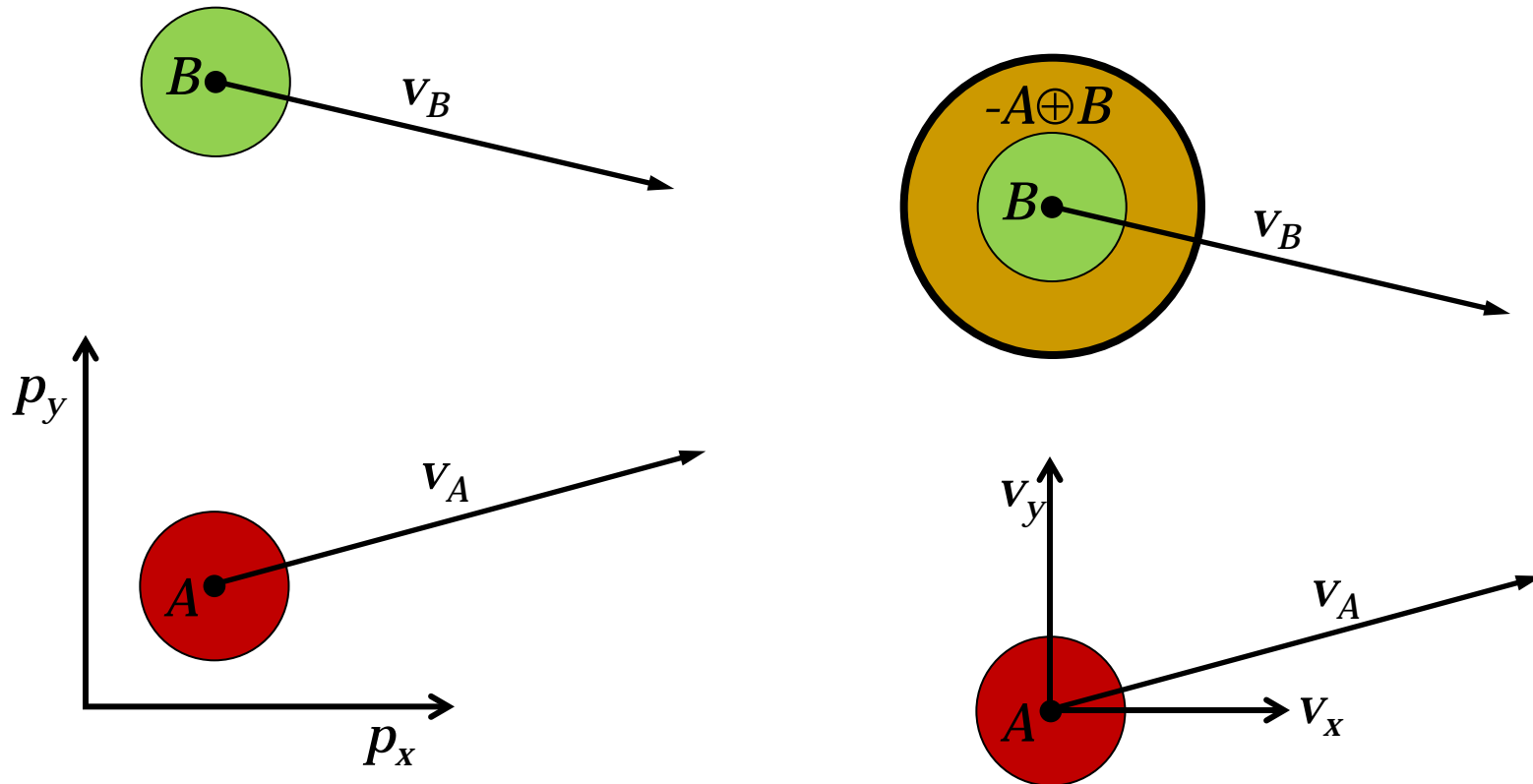
Velocity obstacle method

- Given robots A and B moving in the plane, will they collide in ΔT ?
 - Locate A at the origin of coordinates ($p_A - p_A = 0$) and B at ($p_B - p_A$)



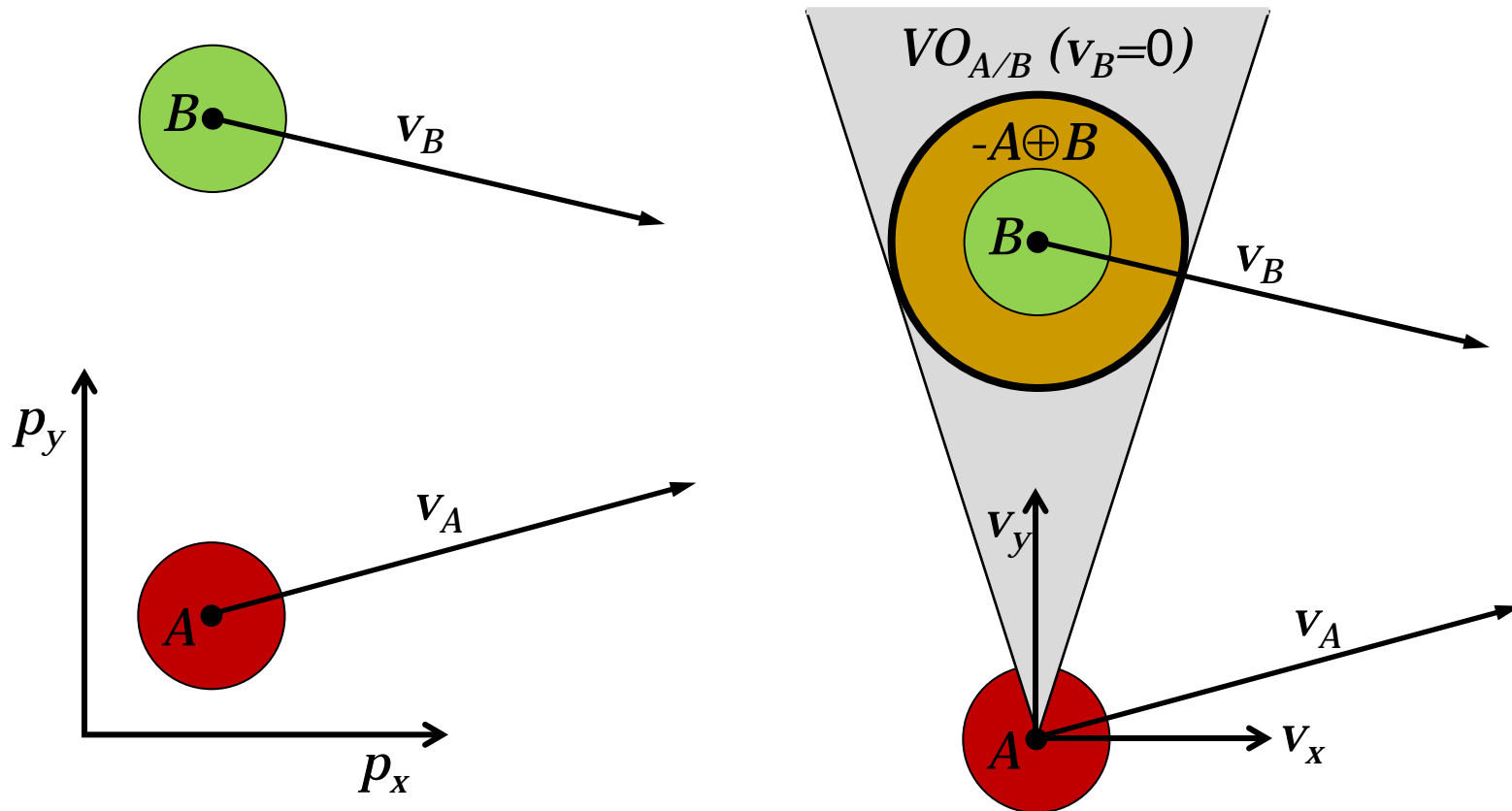
Velocity obstacle method

- Given robots A and B moving in the plane, will they collide in ΔT ?
 - Minkowski difference of coordinates (radius r_A+r_B)



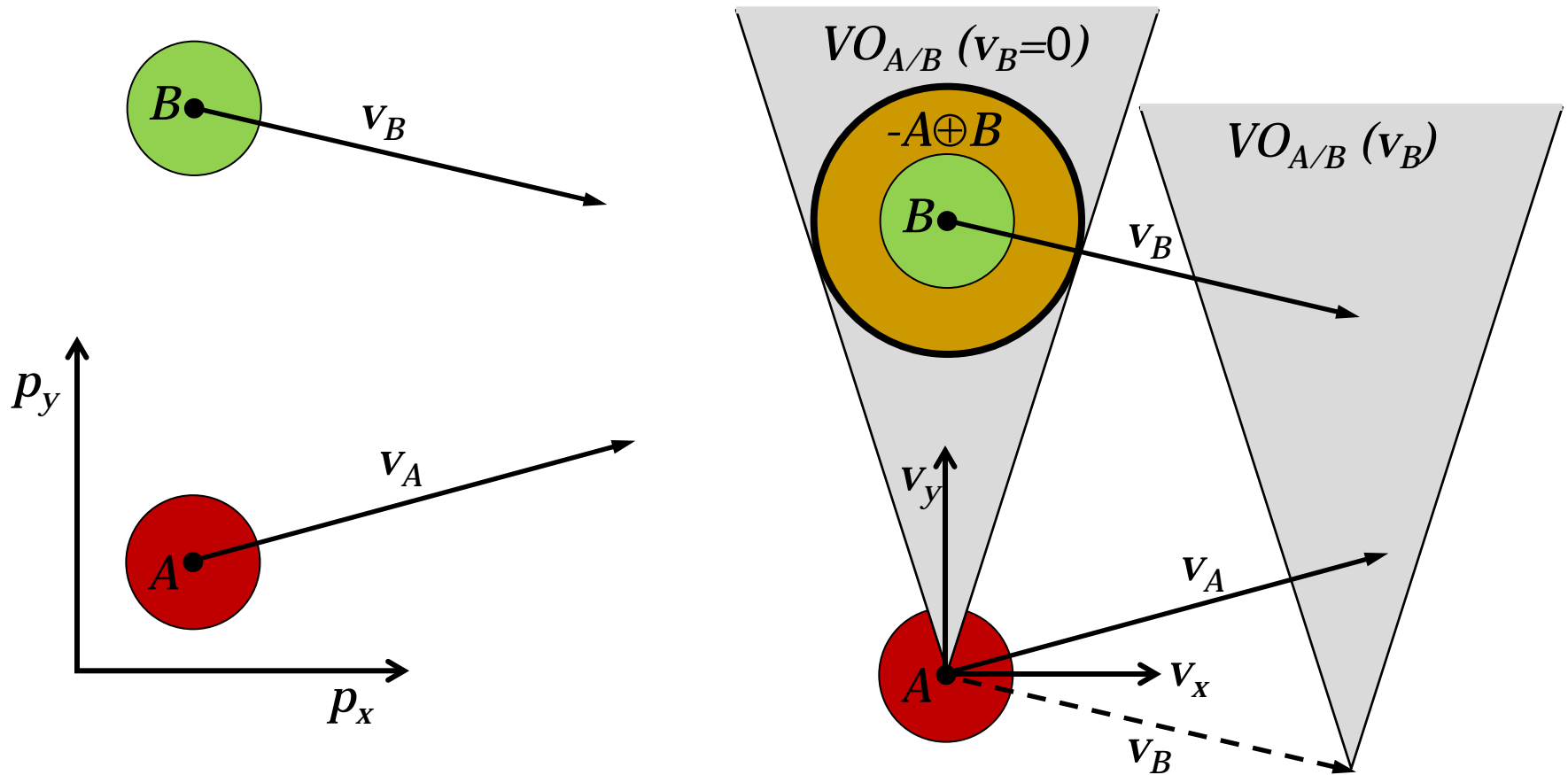
Velocity obstacle method

- Given robots A and B moving in the plane, will they collide in ΔT ?
 - Denote the velocity obstacle for robot A induced by dynamic obstacle B with $VO_{A/B}$



Velocity obstacle method

- Given robots A and B moving in the plane, will they collide in ΔT ?
 - Velocity obstacle $VO_{A/B}$ is the set of velocities of A that will result in collision with B in some time, assuming constant velocities



Velocity obstacle method

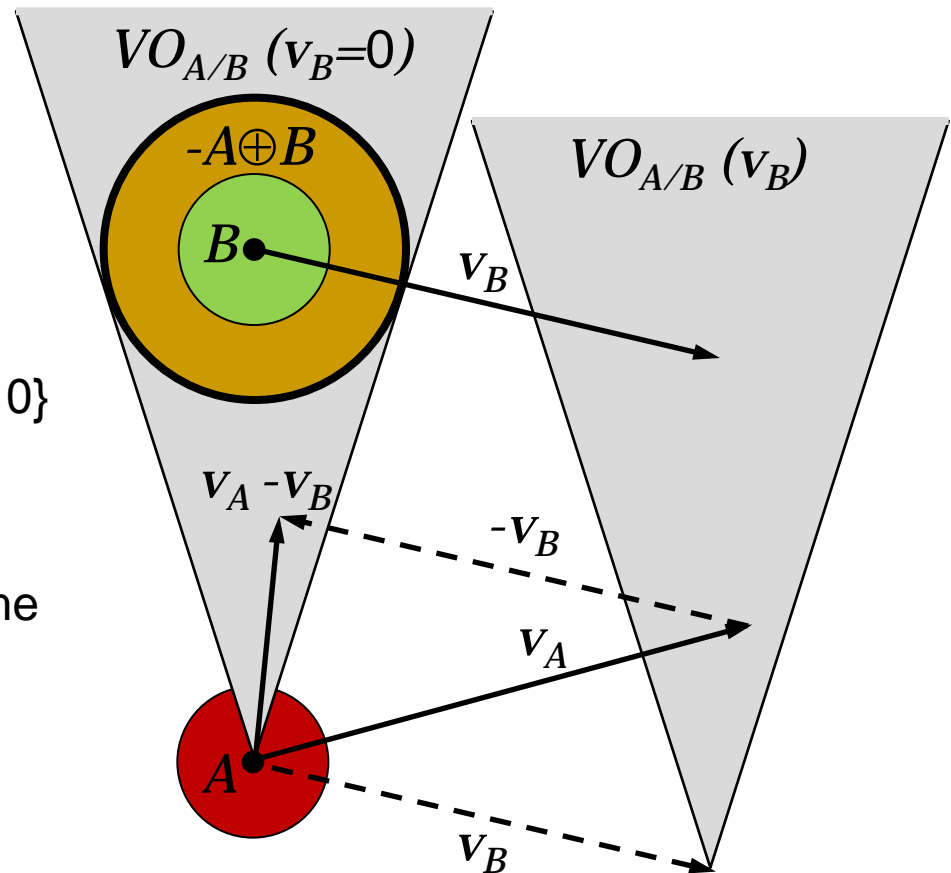
- Given robots A and B moving in the plane, will they collide in ΔT ?
 - Equivalence: if v_A lies in $VO_{A/B}(v_B)$ the relative velocity $v_A - v_B$ lies in the velocity obstacle of B to A , assuming B does not move, i.e. $VO_{A/B}(v_B=0)$

Let us denote $\lambda(p, v)$ a ray from p with direction v :

$$\lambda(p, v) = \{p + t v / t \geq 0\}$$

$$VO_{A/B}(v_B) = \{v_A / \lambda(p_A, v_A - v_B) \cap (-A \oplus B) \neq \emptyset\}$$

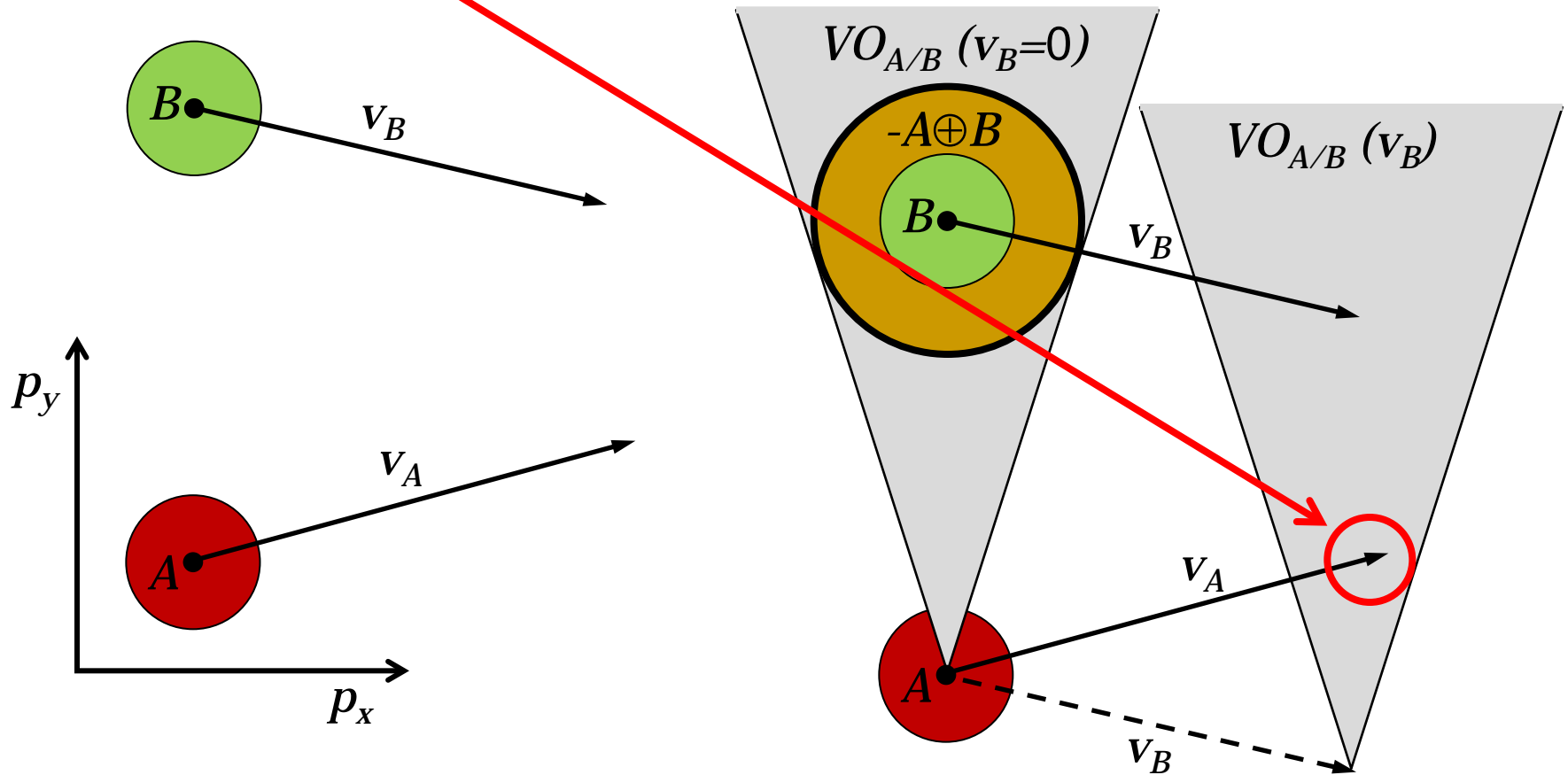
Therefore, if $v_A \in VO_{A/B}(v_B)$
 $\Rightarrow A$ and B will collide at some time
 Otherwise, if $v_A \notin VO_{A/B}(v_B)$
 $\Rightarrow A$ and B will never collide



Velocity obstacle method

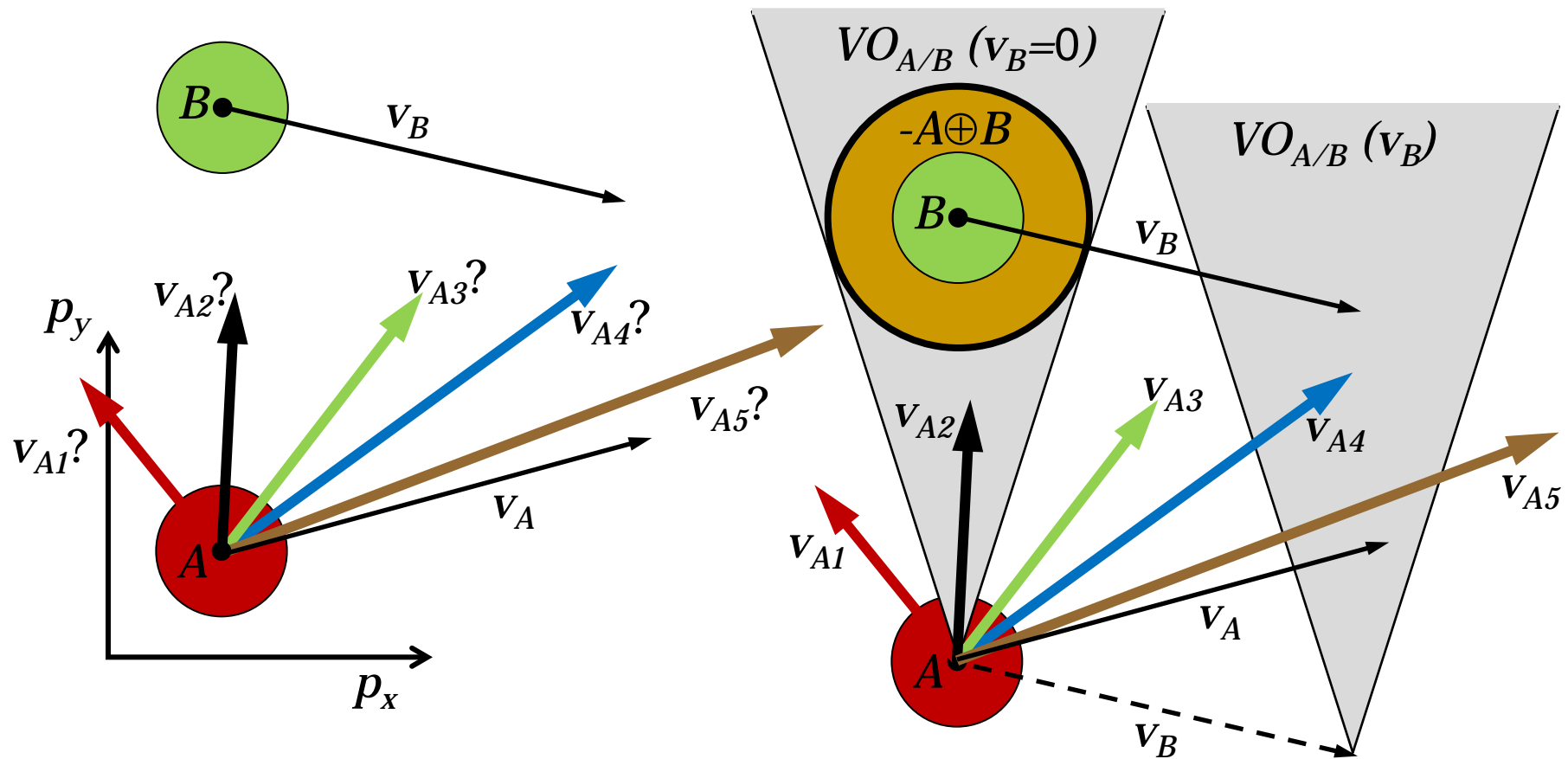
- Given robots A and B moving in the plane, will they collide in ΔT ?

□ Yes



Velocity obstacle method

- Given robots A and B moving in the plane, will they collide in ΔT ?
 - In each step, choose a velocity v_A outside any of the velocity obstacles VO induced by the moving obstacles



Velocity obstacle method

- Any strategy that selects velocities outside all VO will do
- Given a preferred velocity or a goal position, choose from all the free collision velocities the closest to them and the robot will reach safely the goal



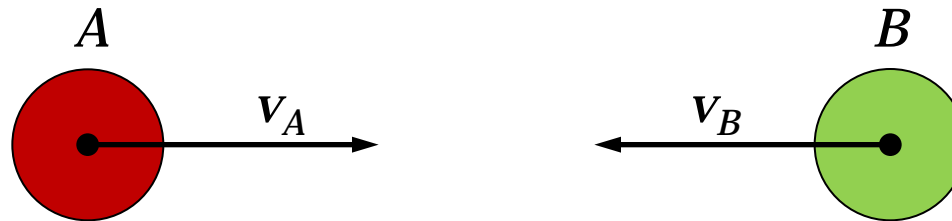
■ Drawbacks

- In multirobot settings, the robots react when they perceive each other, adapting their motion. This should be taken into account
- This may lead to undesired oscillations
- There is also the problem of “reciprocal dances”. Although they may be common in natural human motion, they are undesirable for multi-robot navigation



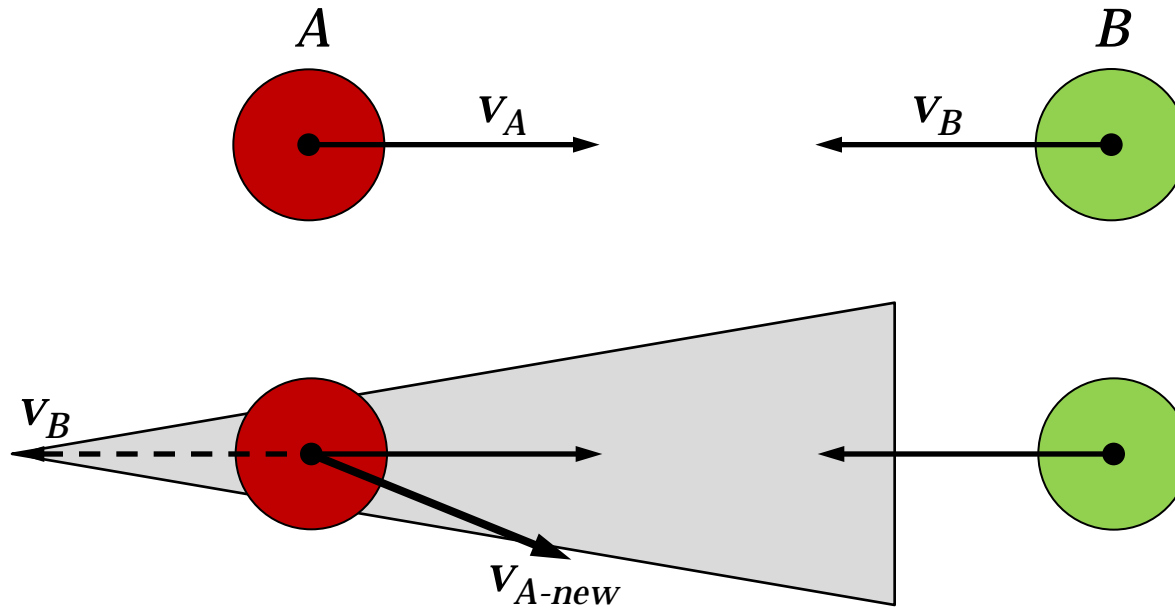
Reciprocal velocity obstacle method

- Oscillation problem of velocity obstacle method



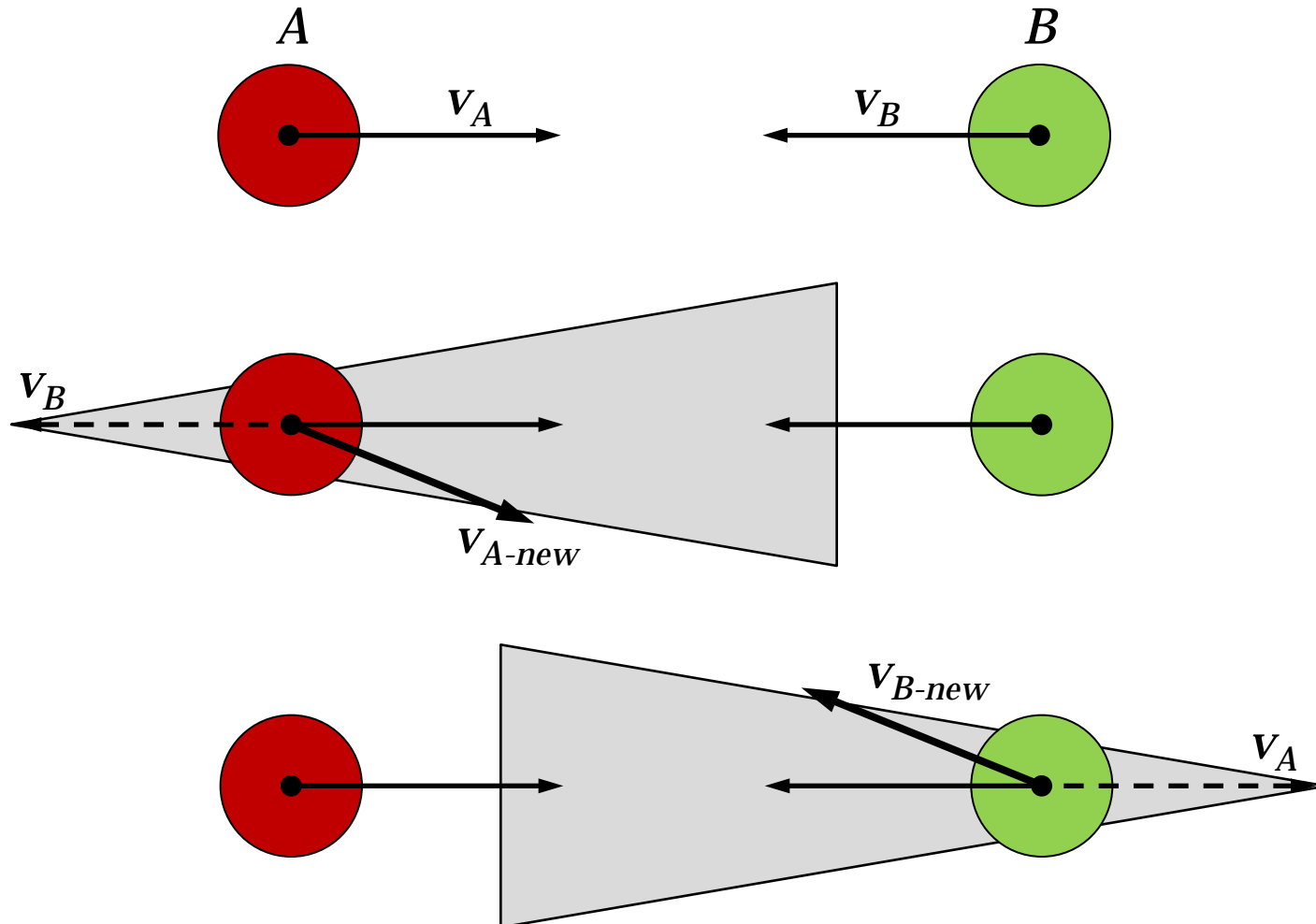
Reciprocal velocity obstacle method

■ Oscillation problem of velocity obstacle method



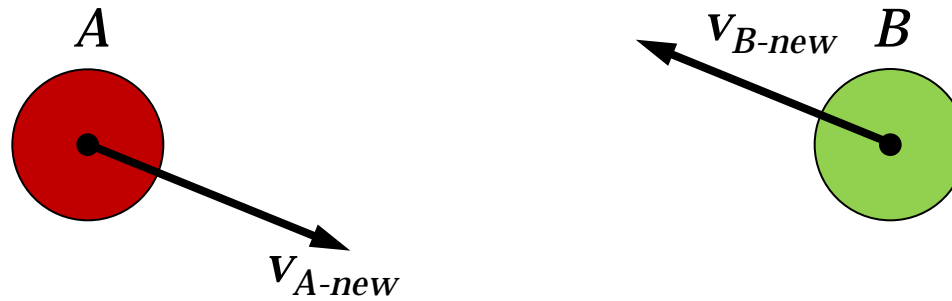
Reciprocal velocity obstacle method

■ Oscillation problem of velocity obstacle method



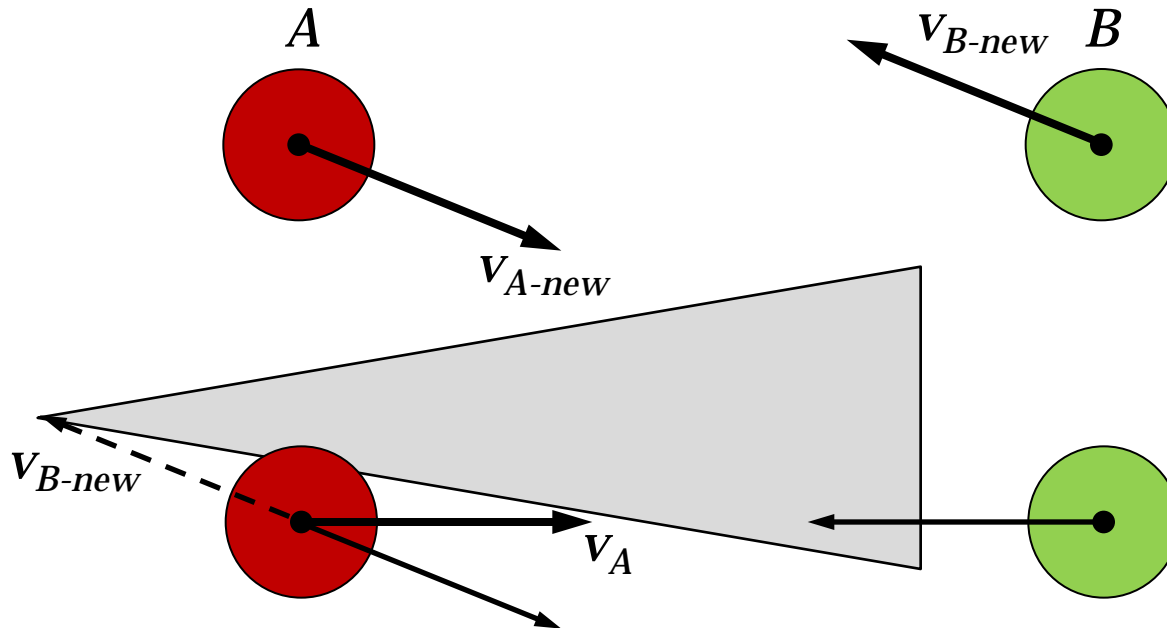
Reciprocal velocity obstacle method

■ Oscillation problem of velocity obstacle method



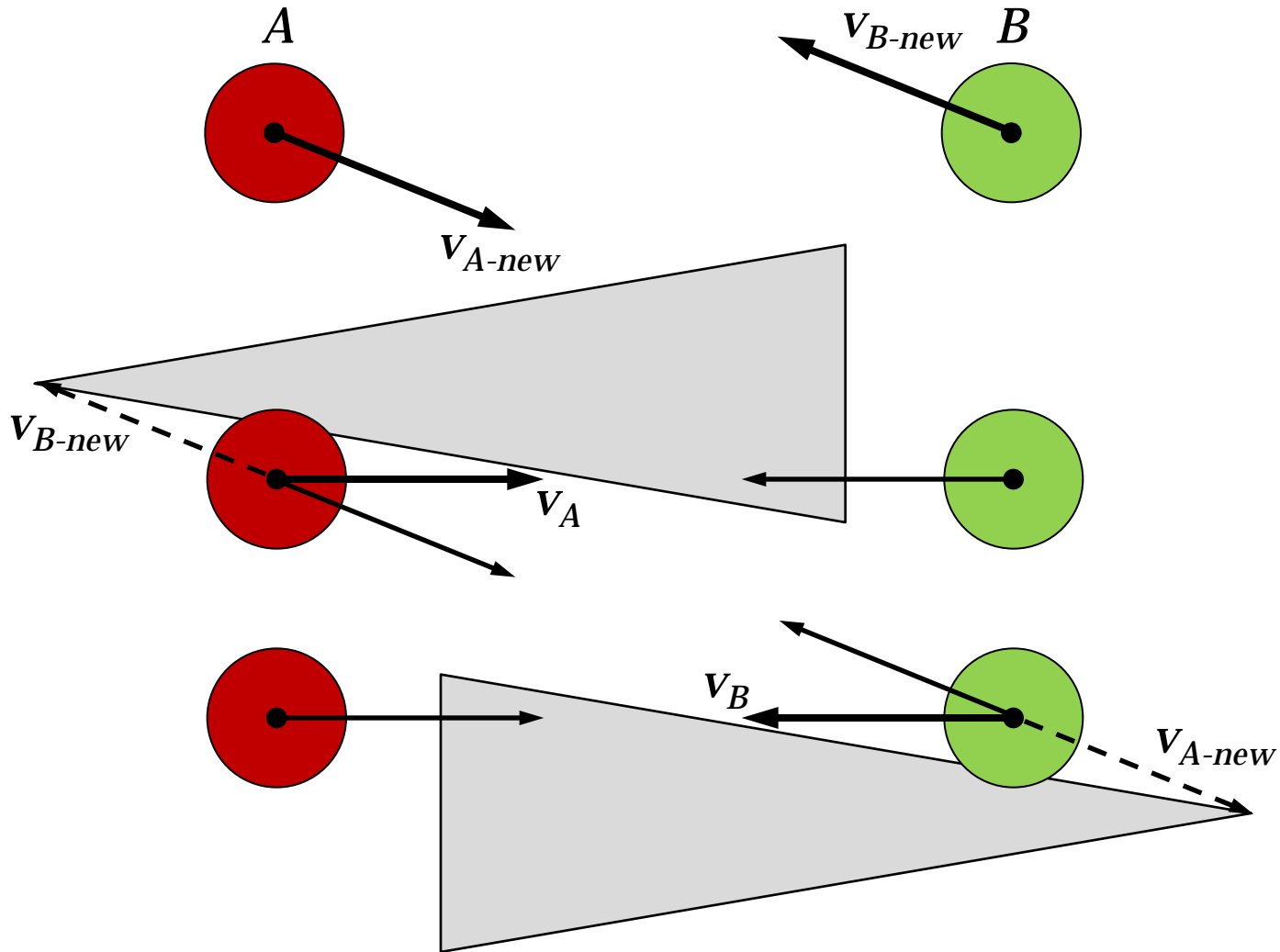
Reciprocal velocity obstacle method

■ Oscillation problem of velocity obstacle method



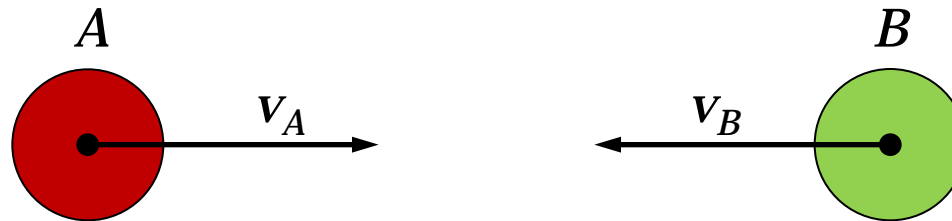
Reciprocal velocity obstacle method

■ Oscillation problem of velocity obstacle method



Reciprocal velocity obstacle method

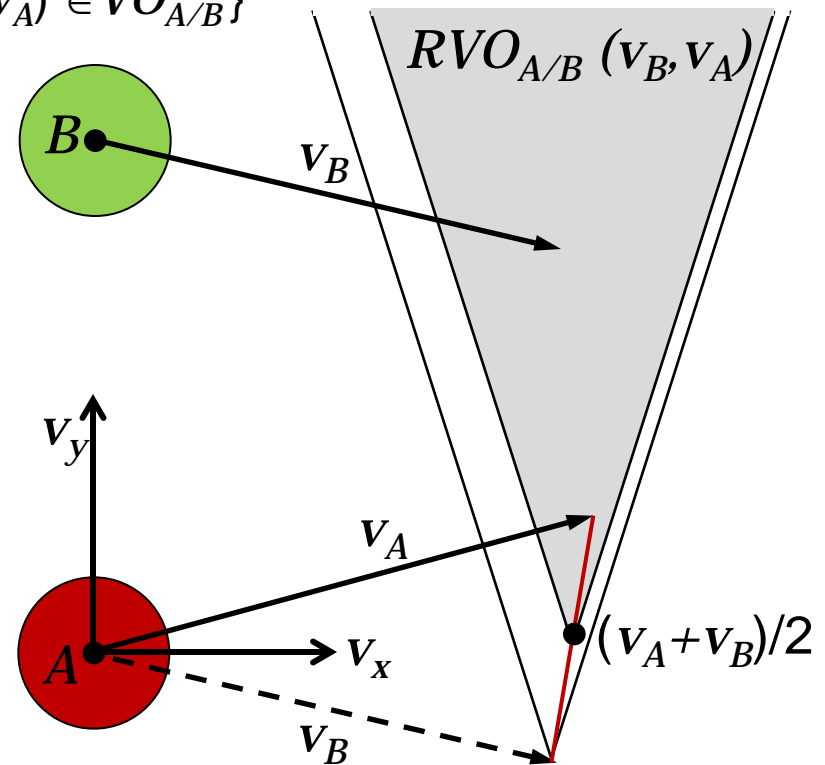
- Oscillation problem of velocity obstacle method



- And history repeats itself...

Reciprocal velocity obstacle method

- The idea of the reciprocal velocity obstacle method (RVO) is:
 - Instead of choosing a new velocity for each agent that is outside the other agent's velocity obstacle,
 - we choose a new velocity that is the average of its current velocity and a velocity that lies outside the other agent's velocity obstacle.
 - $RVO_{A/B}(v_B, v_A) = \{ v \text{ such that } (2v - v_A) \in VO_{A/B} \}$



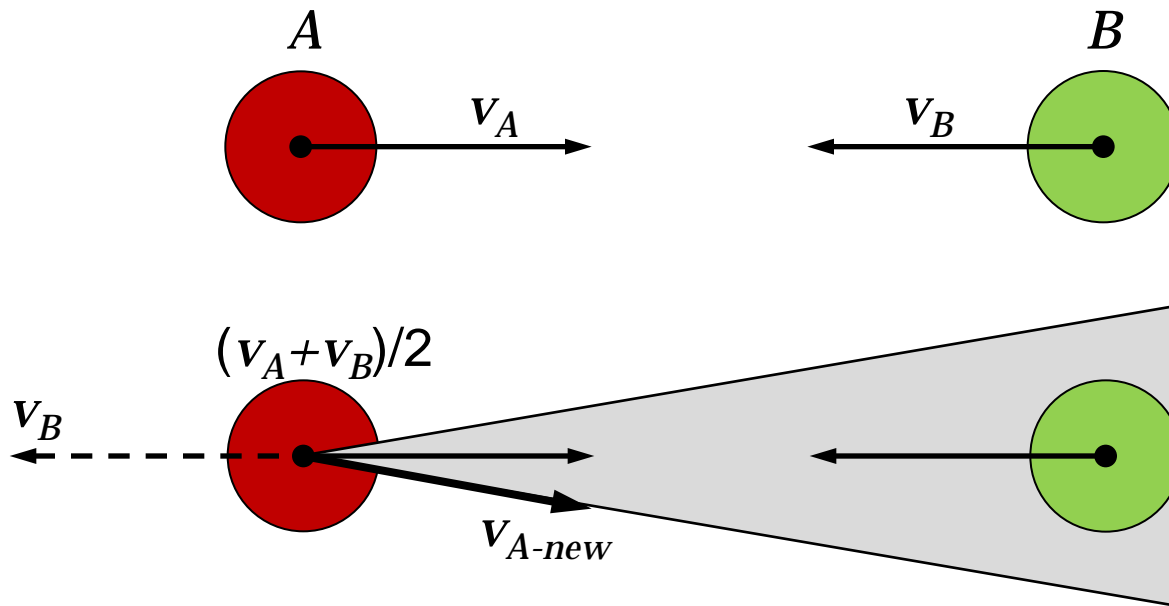
Reciprocal velocity obstacle method

- Let us check the oscillation problem



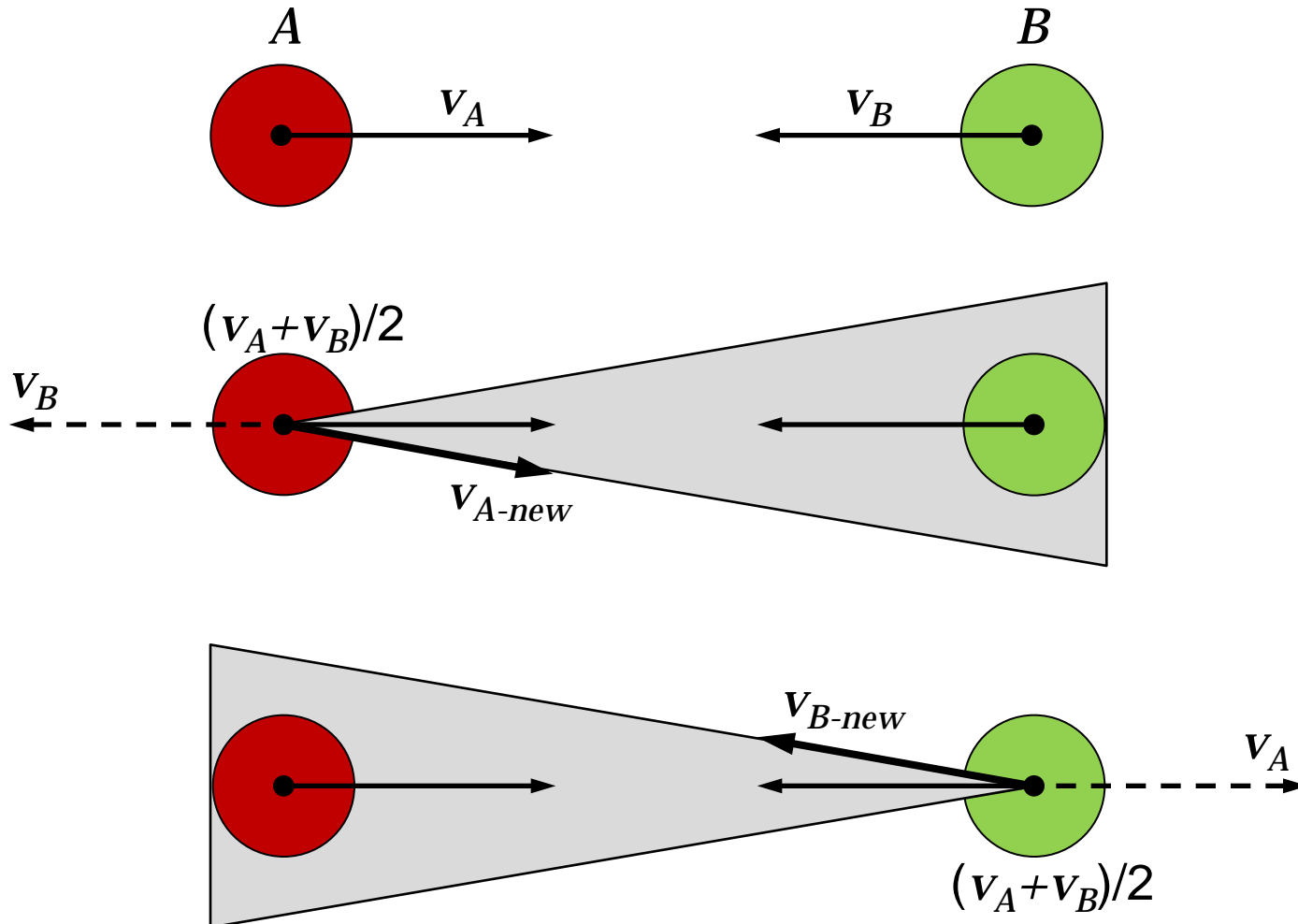
Reciprocal velocity obstacle method

- Let us check the oscillation problem



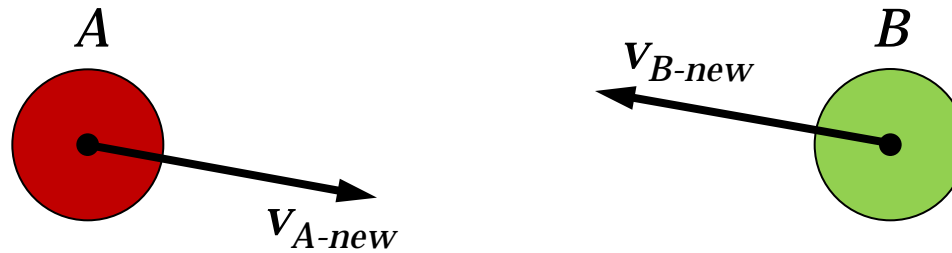
Reciprocal velocity obstacle method

- Let us check the oscillation problem



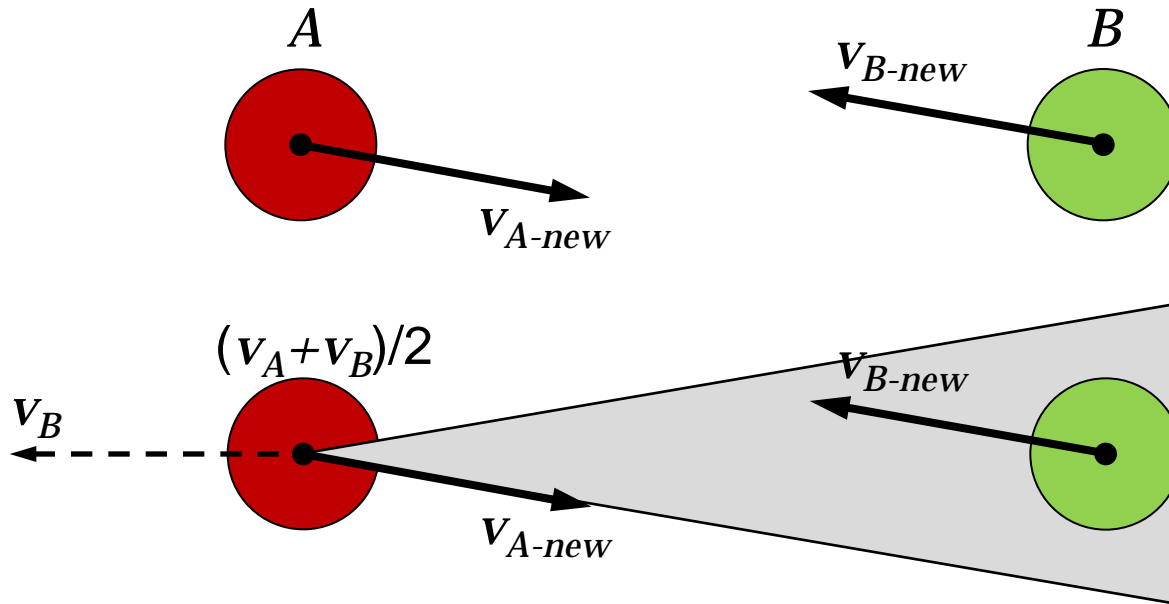
Reciprocal velocity obstacle method

- Let us check the oscillation problem



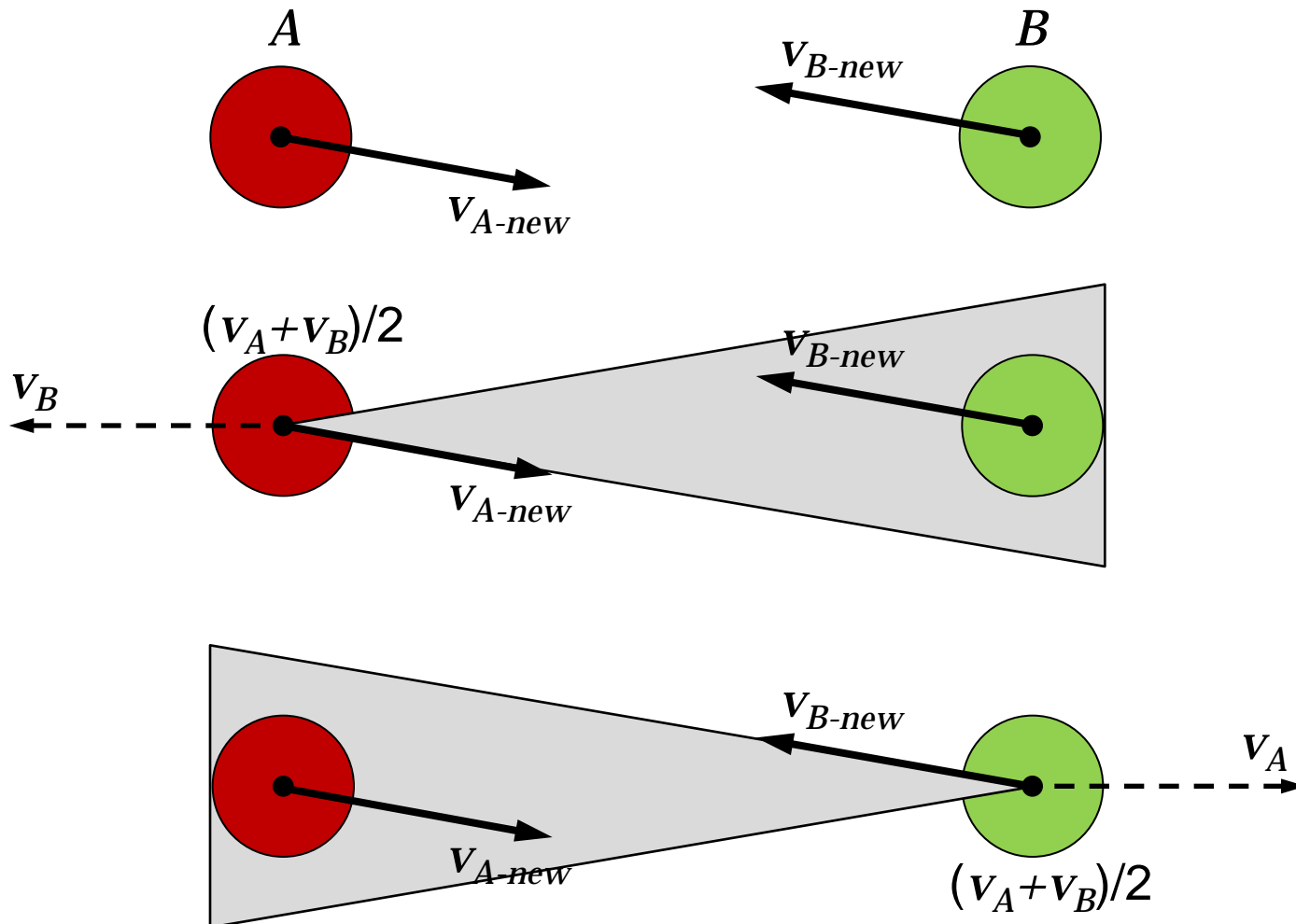
Reciprocal velocity obstacle method

- Let us check the oscillation problem



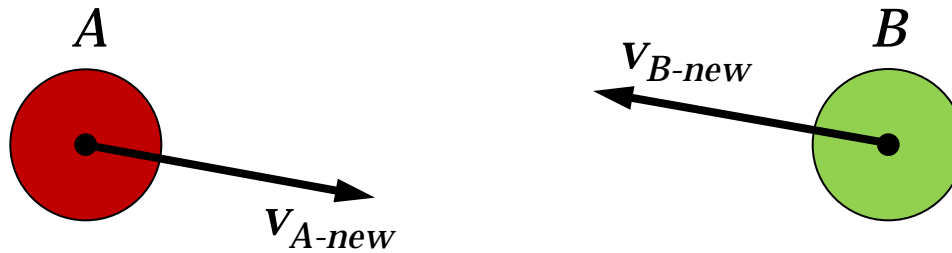
Reciprocal velocity obstacle method

- Let us check the oscillation problem

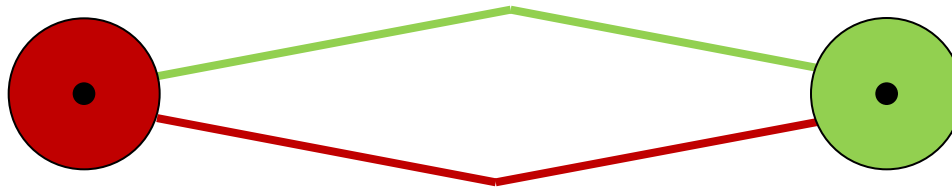


Reciprocal velocity obstacle method

- Let us check the oscillation problem

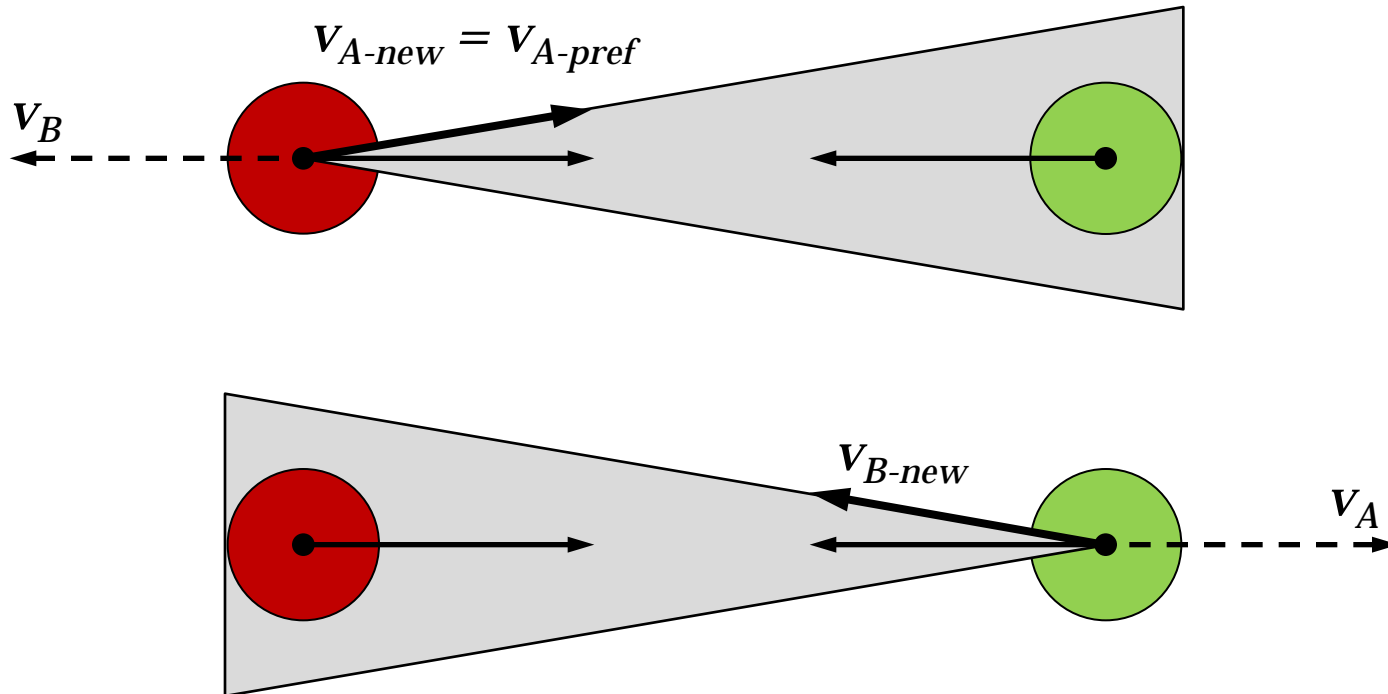


- Motion result:



Reciprocal velocity obstacle method

- However, rather than choosing the velocities closest to their current velocities, usually the robots are required to
 - follow a velocity closest to their preferred velocities,
 - A third robot may also cause to choose a v on the conflicting side

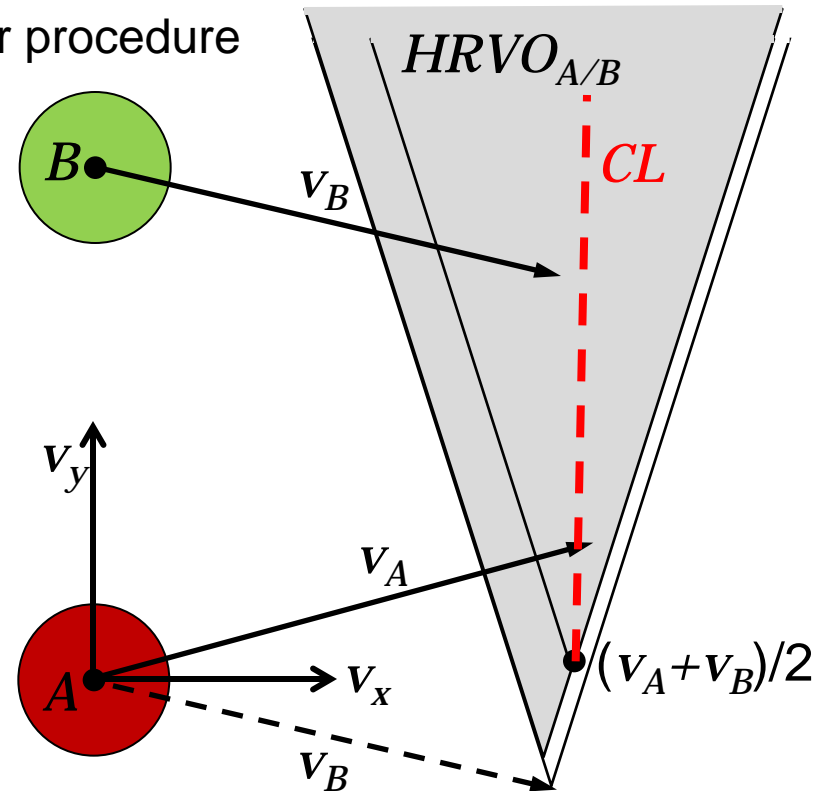


- Therefore, the “reciprocal dances” still may happen

Hybrid reciprocal velocity obstacle method

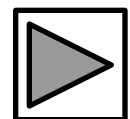
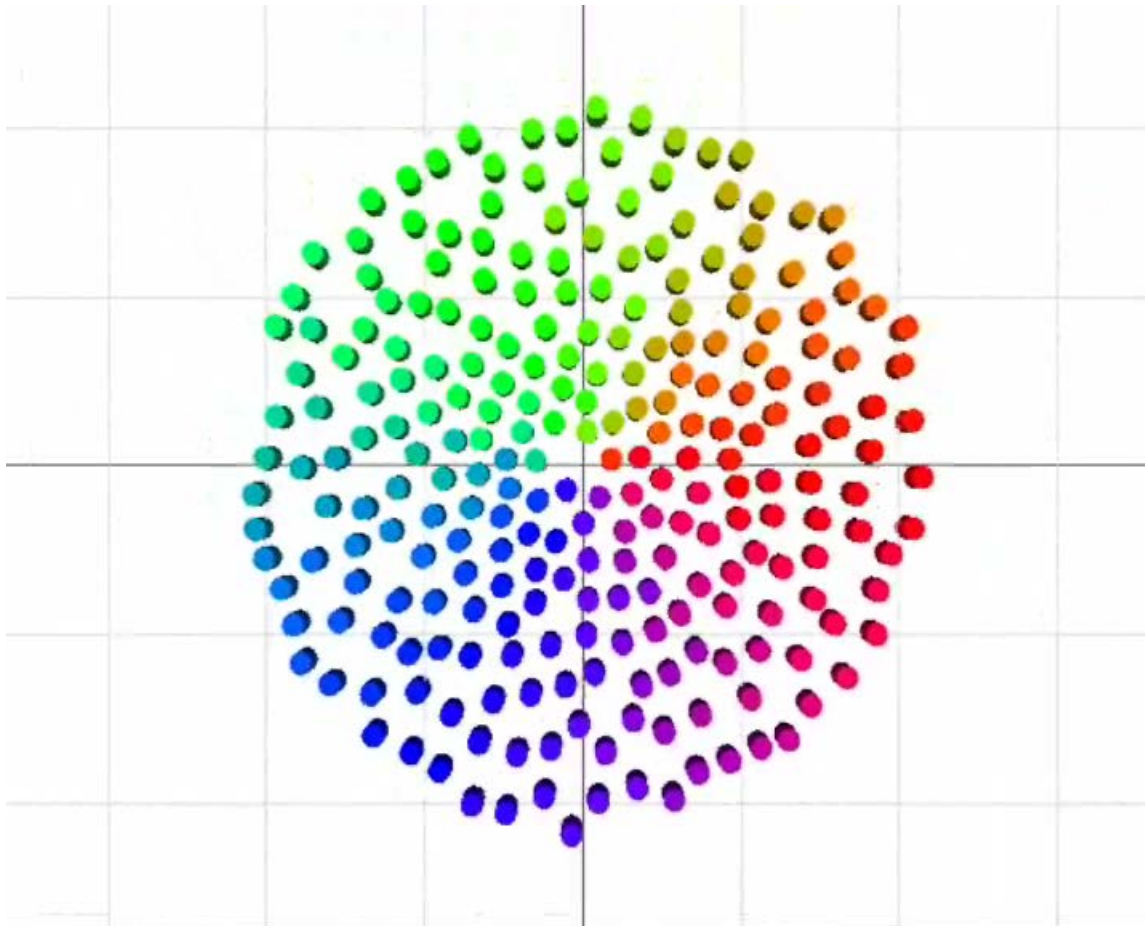
■ HRVO

- We introduce the **Center Line** of $RVO_{A/B}$, **CL**:
- If v_A is to the right of **CL** of $RVO_{A/B}$ (so v_B is to the right of **CL** of $RVO_{B/A}$) we want A to choose v_{A-new} to the right of **CL** of $RVO_{A/B}$
- The $RVO_{A/B}$ is enlarged on the left side => $HRVO_{A/B}$
- If v_A is to the left of **CL** perform mirror procedure
- This greatly reduces the possibility of oscillations



Hybrid reciprocal velocity obstacle method

■ Examples



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- Jamie Snape, Jur P. van den Berg, Stephen J. Guy, Dinesh Manocha. Independent navigation of multiple mobile robots with hybrid reciprocal velocity obstacles. *IROS 2009.* 5917-5922
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