## Syllabus

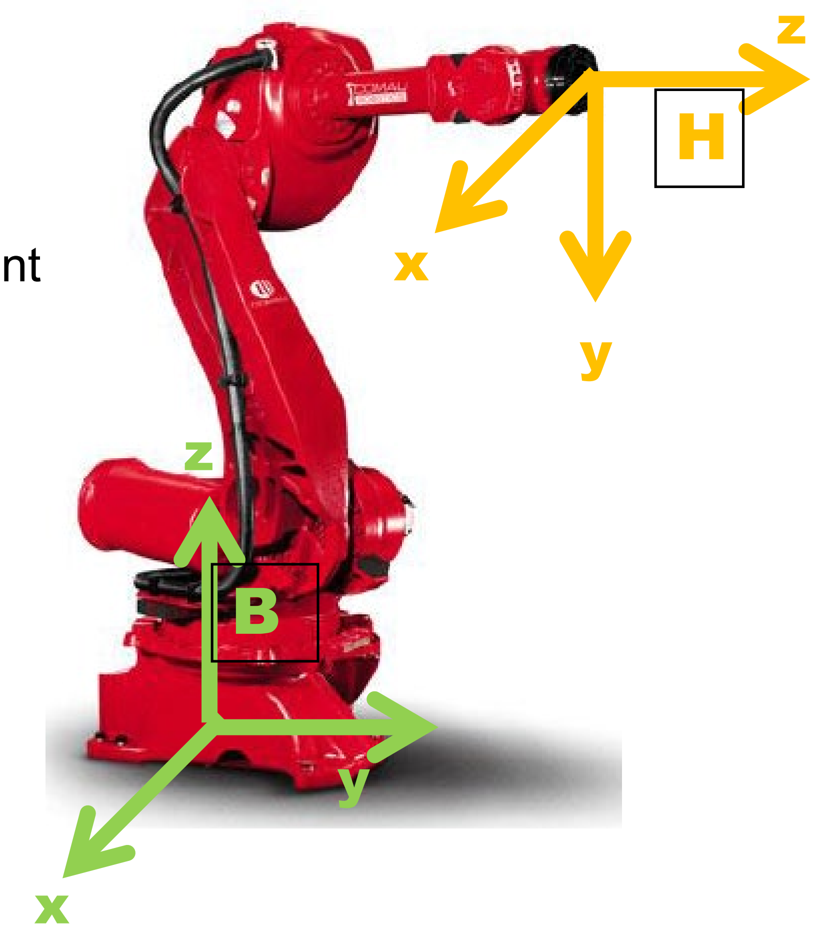
- **Robotic Manipulators**
  - Applications in assistive robotics: medical robotics, prostheses, companion, monitorization, assistance
  - Structure of the robot manipulators
  - Drives and sensors
  - Spatial location: transformations
  - Geometric model
  - Motion control

- **Visual Servoing**
  - Position-Based Visual Servoing. 3D scene model, two-view geometry, stereo cameras. Control laws.
  - Image-Based Visual Servoing. Definition of targets. Interaction matrix. Image-based visual control loop.
  - Stability analysis. Lyapunov.
  - Hybrid Visual Servoing.

- **Forward Geometric Model:** Where is G given $q_i$ ?

  - Forward model (geometric, forward kinematic model)
  - Denavit-Hartenberg (DH) parameters
  - Computation of forward model from joint coordinates

- **Inverse Geometric Model:** What are the joint coordinates $q_i$ given G?
  - Inverse model
  - Redundant solutions and singularities
  - Analytical and numerical resolution methods



## Geometric Model

- Mechanical structure:
  - n links or solid rigid bodies L1,...Ln (open loop chain)
  - n joints or rotational or prismatic joints J1,..,Jn

  **GOAL:** To place the last link at a desired location in the environment (position and orientation)

- Description of the geometry
  - Joint coordinates $\mathbf{q} = (q_1, \ldots, q_n)^t$
  - Cartesian coordinates (task, operational) $\mathbf{X} = (x_1, \ldots, x_m)^t$

$$\mathbf{X}(q) \leftrightarrow {}^0\mathsf{T}_6(\mathbf{q});$$

# Forward geometric model

- Kinematic chain with links connected by joints R or P
  - To assign a reference frame to every link

$$^{0}\mathbf{T}_n = {}^{0}\mathbf{T}_1 \; {}^{1}\mathbf{T}_2 \; \ldots \; {}^{n-1}\mathbf{T}_n$$

- Standard representation of the transformation matrices between consecutive spatial links (Denavit and Hartenberg-1955)
- Reference frames: located so that only 4 parameters are needed: $(\theta_i, d_i, a_i, \alpha_i)$

$$^{i-1}\mathbf{T}_i = \mathrm{Rot}(z, \theta_i) * \mathrm{Trasl}(z, d_i) * \mathrm{Trasl}(x, a_i) * \mathrm{Rot}(x, \alpha_i)$$

- The value of these four parameters depends on the geometry of the consecutive links and the value of the joint that connects them (rotational or prismatic joint).
- Fixed / dependent on the current joint position.

https://en.wikipedia.org/wiki/Denavit%E2%80%93Hartenberg_parameters

# Forward geometric model

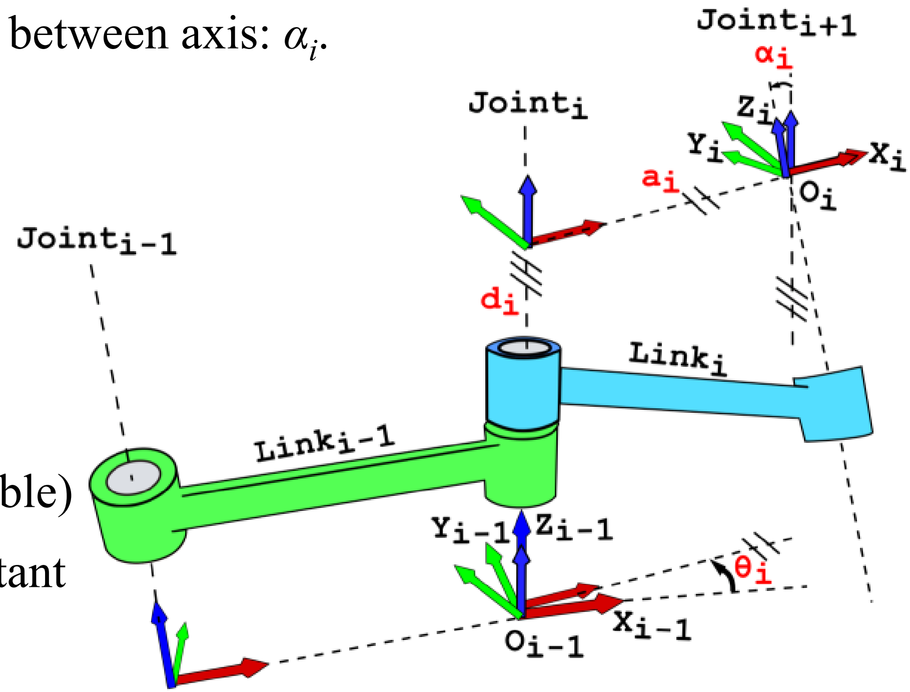
- Every link $i$ is connected, at most, with two links: $i$-$1$ , $i+1$

- Link → connecting two axis
  - Distance between axis: $a_i$.
  - Angle between axis: $\alpha_i$.

- Joint
  - translation: $d_i$.
  - rotation: $\theta_i$.

R joint: $d_i$ constant and $\theta_i$ joint value (variable)

P joint: $d_i$ joint value (variable) and $\theta_i$ constant



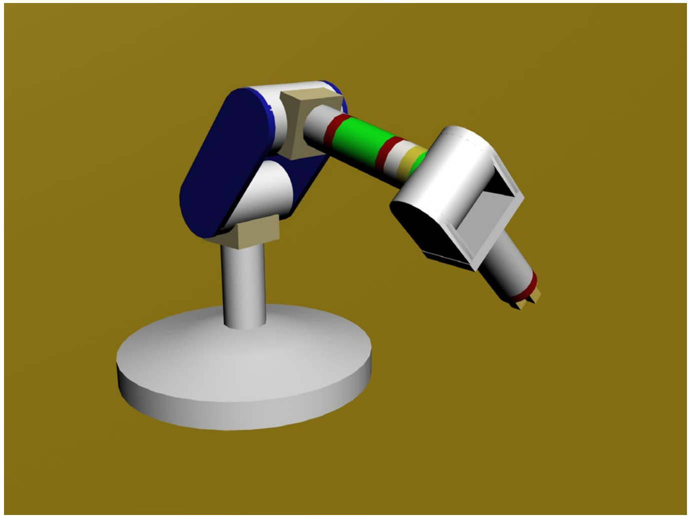
# Forward kinematics

- Use the forward geometric model (Denavit Hartemberg parameters)

$$^{i-1}\mathbf{T}_i = \mathrm{Rot}(\mathbf{z}_{i-1}, \theta_i)\ \mathrm{Trasl}(0,0,d_i)\ \mathrm{Trasl}(a_i,0,0)\ \mathrm{Rot}(\mathbf{x}_{i-1}, \alpha_i)$$

| | θi | di | ai | αi |
|---|---|---|---|---|
| **1** | 30+90+q1 | 0 | 0 | 90 |
| **2** | 0 | L1+q2 | 0 | 0 |

- Apply the values for the joint coordinates q1, q2 …
- Obtain the resulting pose of the robot tool.



# Inverse geometric model

$$\mathbf{q} = \left(q_1, q_2, q_3, \cdots, q_n\right)^T$$

$$\mathbf{X} = \left(x, y, z, \phi, \theta, \psi\right)^T \leftrightarrow {}^0\mathbf{T}_n$$

$$\mathbf{q} = g^{-1}(\mathbf{X})$$

- Robot configurations. redundant solutions
- Inverse geometric solutions:
  - Numerical
  - Analytical
- It does not have a unique solution, or a systematic process to obtain it

# Inverse Geometric Model: Numerical solvers

- Iterative, from an initial seed
- Linearization of the geometric model

$$\mathbf{X} = \mathbf{g}(\mathbf{q}) \quad \longleftarrow \text{Forward model}$$

$$\mathbf{dX} = \frac{\partial \mathbf{g}}{\partial \mathbf{q}} \Big|_0 \, \mathbf{dq} = \mathbf{J}(\mathbf{q}) \Big|_0 \, \mathbf{dq}$$

$$\rightarrow \Delta\mathbf{X} = \mathbf{J}(\mathbf{q}) \Big|_0 \, \Delta\mathbf{q}$$

- **J(q)** is the **Jacobian** of the robot (partial derivatives of the forward model relative to q)

$$\mathbf{T} = \mathrm{hom}(\mathbf{X})$$

$$\mathbf{X} = \mathrm{loc}(\mathbf{T})$$

```
inverse_model(wXf , q0):
  wTf = hom(wXf)                        ← Goal pose
  q = q0
  while | Δq | > ε
                                        ← Current pose
     wTk = hom( forward_model(q) )
     ΔX = loc ( kTw * wTf)
     compute J(q)
     Δq = J(q)^-1 ΔX                    ← Desired relative displacement from k to goal
     q = q + Δq
  end while
 return q
```

# Singularity

- Location in which some Cartesian degree of freedom is lost
- Near the limits of the workspace / robot envelope.
- Arm fully extended or folded
- Within the robot's work envelope:
  - Points where there is a change of the robot configuration
  - Usually, two or more joint axis aligned
- Impossible to move smoothly following some direction

Forward model

$$\mathbf{X} = \mathbf{g}(\mathbf{q})$$

$$\mathbf{dX} = \frac{\partial \mathbf{g}}{\partial \mathbf{q}}\Big|_0 \mathbf{dq} = \mathbf{J}(\mathbf{q})\big|_0 \mathbf{dq}$$

$$\rightarrow \Delta\mathbf{X} = \mathbf{J}(\mathbf{q})\big|_0 \, \Delta\mathbf{q}$$

$$|\mathbf{J}(\mathbf{q})| = 0$$

- Inverse geometric model
  - There may exist infinite solutions -> to avoid indeterminations, random solutions, numerical problems
  - Generation of Cartesian trajectories -> to be aware of singularities!

# Inverse Geometric Model: Analytical solvers

- To decompose the spatial geometry into planar sections + to solve triangles
- Multiple solutions can be obtained starting from the first one
  - Signs of and origin of triangles
  - Angle quadrants

Wrist-partitioned robots: Kinematic decoupling arm (translation) & wrist (rotation)

- Sufficient condition (DH) $a_4 = a_5 = d_5 = 0$

$$^0\mathbf{T}_6 = {^0\mathbf{T}_1} * {^1\mathbf{T}_2} * {^2\mathbf{T}_3} * \mathrm{Trasl}(0,0,d_4) *$$

$\mathbf{T}_{P123}(q_1, q_2, q_3)$

$$\mathrm{Rot}(z,\theta_4) * \mathrm{Rot}(x,\alpha_4) * \mathrm{Rot}(z,\theta_5) * \mathrm{Rot}(x,\alpha_5) * \mathrm{Rot}(z,\theta_6) *$$

$$\mathrm{Trasl}(0,0,d_6) * \mathrm{Trasl}(a_6,0,0) * \mathrm{Rot}(x,\alpha_6)$$

$\mathbf{T}_{\mathbf{R456}}$ pure rotation

$^{\mathbf{M}}\mathbf{T}_{\mathbf{6}}$ constant

Since $\mathbf{T}_{\mathbf{R456}}$ pure rotation, position component of $^{\mathbf{0}}\mathbf{T}_{\mathbf{M}}$ and $\mathbf{T}_{\mathbf{P123}}$ are equal

$\mathbf{T}_{R456}(q_4, q_5, q_6)$

$$^3\mathbf{T}_M = \mathbf{T}_{R456} = \mathbf{T}_{P123}^{-1} * \mathbf{V}_0 * {^M\mathbf{T}_6}^{-1}$$

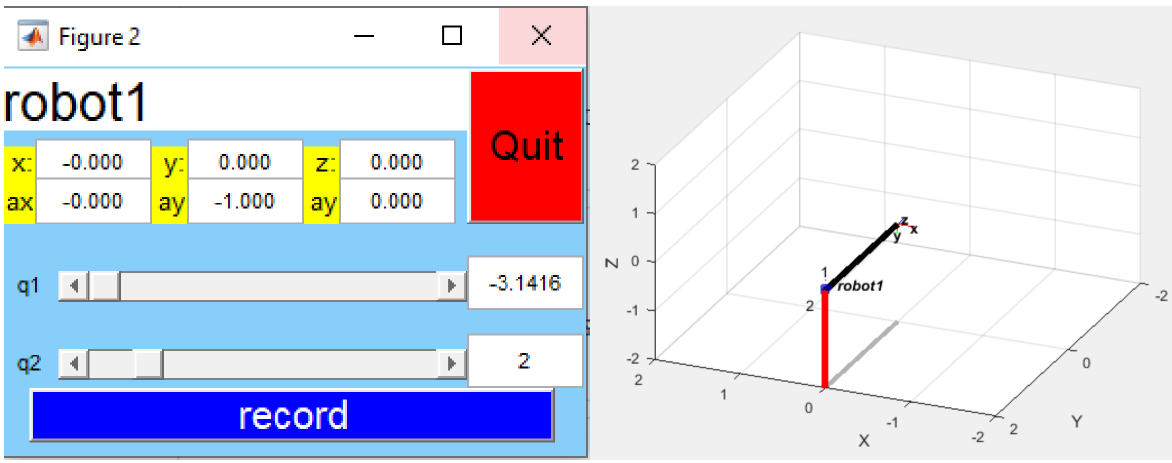
# Geometric model. Exercise

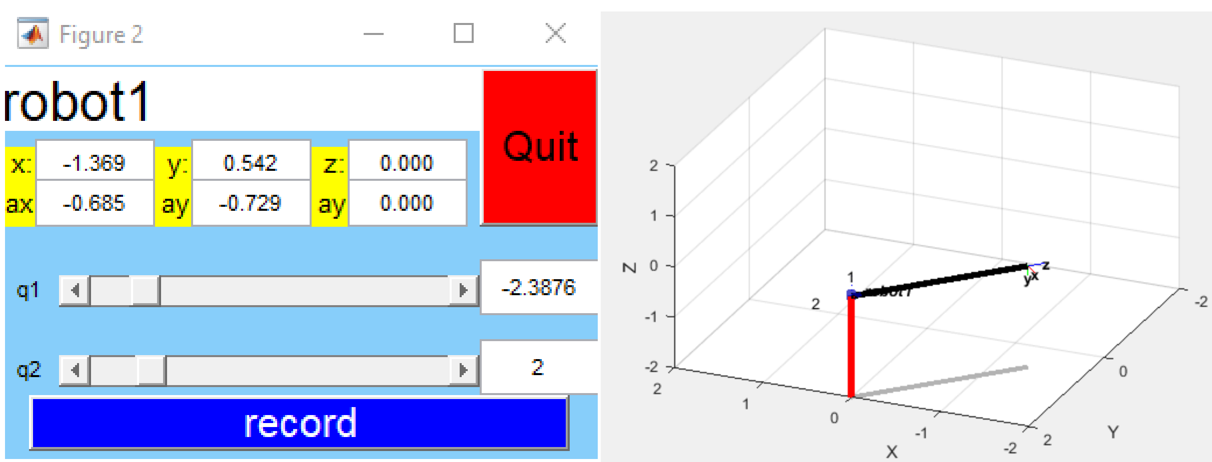
- Run **rtbdemo** at MATLAB
- Pay attention to the instructions for defining the forward and inverse geometric models for a robot, using its Denavit–Hartenberg parameters
- Learn how to use the commands robot.plot() and robot.teach()
- Obtain the DH parameters for the following robot. Implement itm in MATLAB and check that their behavior is the one you would expect.

- **Robot RT**: a robot with two joints. The first one is rotational, and the second one is translational. Next, we show several different positions of the robot. Please, note the associated joint values.

# Geometric model. Exercise

- **Robot RT**



# Geometric model. Exercise

- **Robot RT**