



# Assistive Robotics

## Visual Servoing

Área de Ingeniería de Sistemas y Automática  
Departamento de Informática e Ingeniería de Sistemas  
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## VISUAL SERVOING

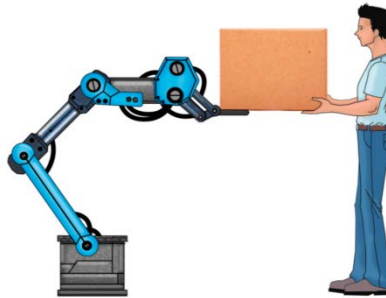
- Introduction
- Robotic manipulation
- Vision systems
- Classification of visual control systems
  - Position-Based Visual Servoing
  - Image-Based Visual Servoing
    - Stability analysis
  - Hybrid Visual Servoing
- Conclusion and bibliography

- **Introduction**
  - **Assistive robotics**
  - **Visual servoing**
- **Robotic manipulation**
- **Vision systems**
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# Introduction: Assistive robotics



# Introduction: Assistive robotics





# Introduction: Visual Servoing

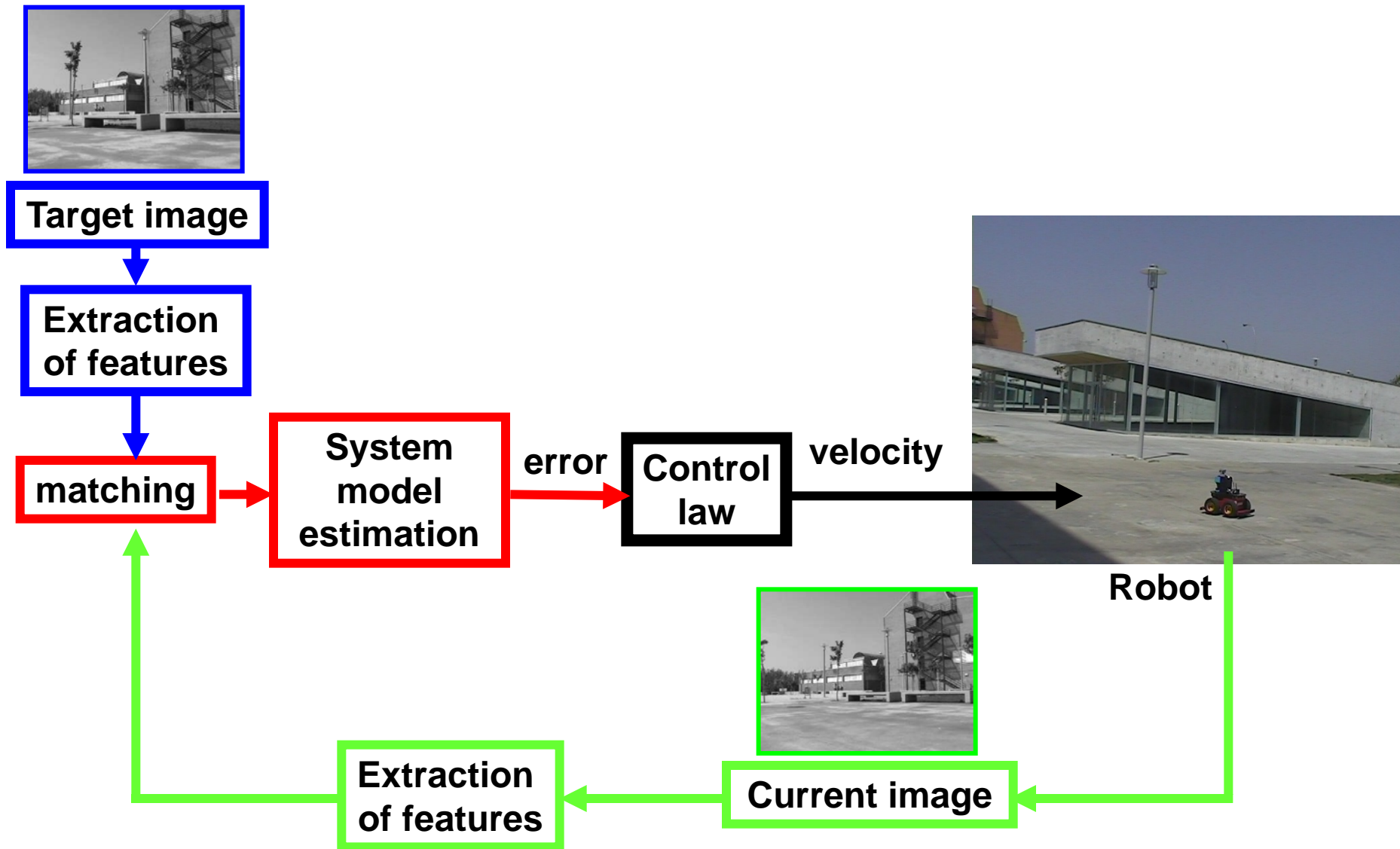
## Open loop



## Closed loop



# Introduction: Visual Servoing



# Introduction: Visual Servoing

- Visual servoing in the market
  - Robotic systems have contributed substantially to increasing the accuracy and speed of automated processes mainly in the manufacturing industry.
  - They generally require a detailed description of the workspace and the objects handled.
    - It requires considerable human and financial resources
  - There is a lack in the perception capability of current robotic systems in terms of:
    - Unknown or dynamic environments, undefined locations, calibration errors, etc.
    - The irruption of visual control responds to these challenges
  - Visual servoing:
    - The objective is the positioning of a robot's end-effector in an unstructured environment.





- Introduction
- **Robotic manipulation**
  - Basics
  - Spatial localization
  - Kinematic model
- Vision systems
- Classification of visual control systems
  - Position-Based Visual Servoing
  - Image-Based Visual Servoing
    - Stability analysis
  - Hybrid Visual Servoing
- Conclusion and bibliography

# Robotic manipulation

## ■ Types of robots

### □ Mobile robots

#### ➤ Terrestrial

- ❖ Wheeled vehicles, crawlers, ...
- ❖ Humanoids, legged

#### ➤ Aerial

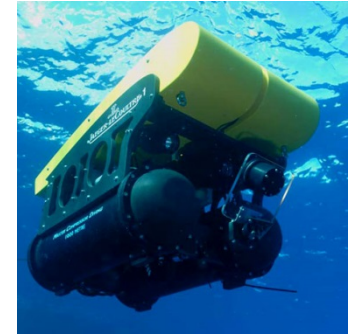
- ❖ Drones, quadrotors, ...

#### ➤ Submarines

### □ Robotic manipulators

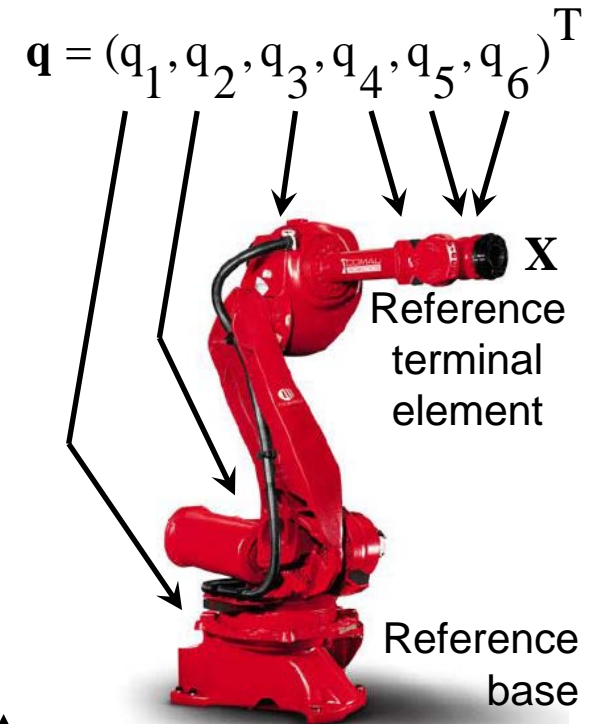
#### ➤ Industrial robots

#### ➤ Collaborative robots



# Robotic manipulation: Spatial localization

- Geometric model to locate the terminal element (position and orientation).
- Joint coordinates:  $\mathbf{q} = (q_1, q_2, \dots, q_n)^T$ 
  - With  $n$  the number of links and joints.
- Coordinates in the task space, operational or Cartesian.  $\mathbf{X} = (x, y, z, rx, ry, rz)^T$
- Reference systems and transformations
  - Reference system associated to each link of the robot.
  - Obtain location from other relative locations

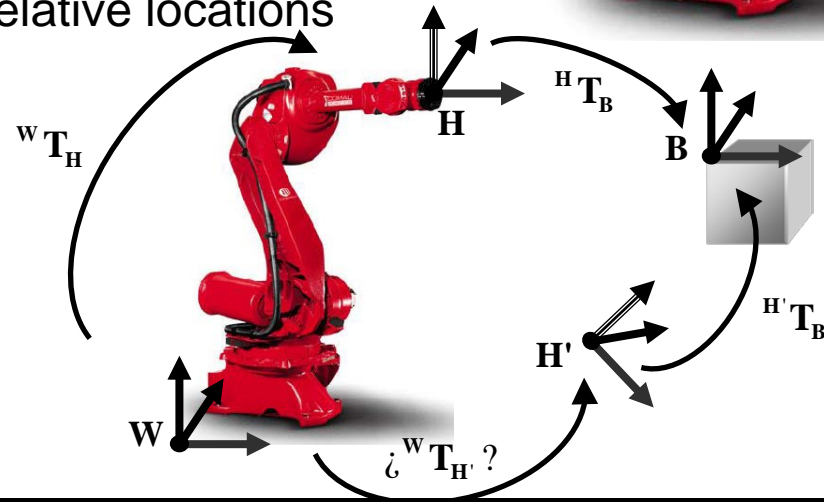


$${}^w\mathbf{T}_H = \begin{pmatrix} \mathbf{R} & \mathbf{p} \\ \mathbf{0} & 1 \end{pmatrix}$$

$${}^i {}^w\mathbf{T}_{H'} ?$$

$${}^w\mathbf{T}_H \cdot {}^H\mathbf{T}_B = {}^w\mathbf{T}_{H'} \cdot {}^{H'}\mathbf{T}_B$$

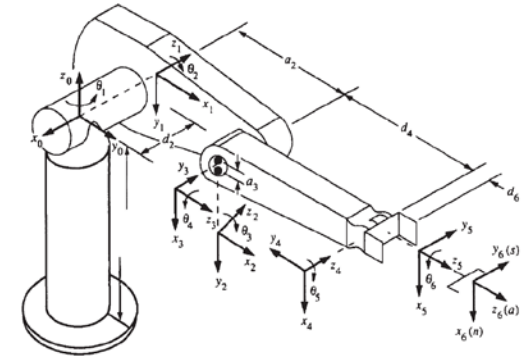
$${}^w\mathbf{T}_{H'} = {}^w\mathbf{T}_H \cdot {}^H\mathbf{T}_B \cdot ({}^{H'}\mathbf{T}_B)^{-1}$$



# Robotic manipulation: Kinematic model

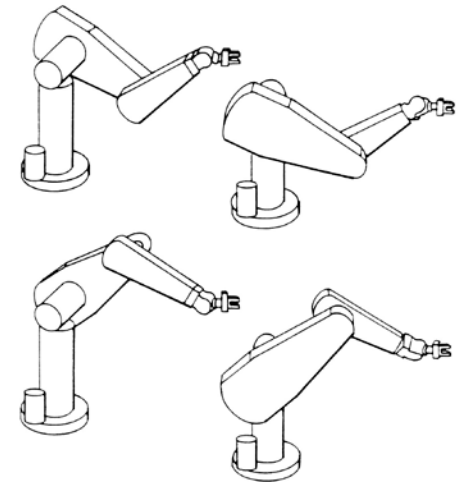
## ■ Direct geometric model: $\mathbf{X} = f(\mathbf{q})$

- ❑ Motion analysis (compute  $\mathbf{X}$  from  $\mathbf{q}$ ):
- ❑ Denavit-Hartenberg (1955) reference assignment.
  - Kinematic chain of links connected with joints.
  - Association of a reference system to each link
$${}^0\mathbf{T}_n = {}^0\mathbf{T}_1 {}^1\mathbf{T}_2 \cdots {}^{n-1}\mathbf{T}_n$$
  - Normalized representation of transformations between consecutive links.



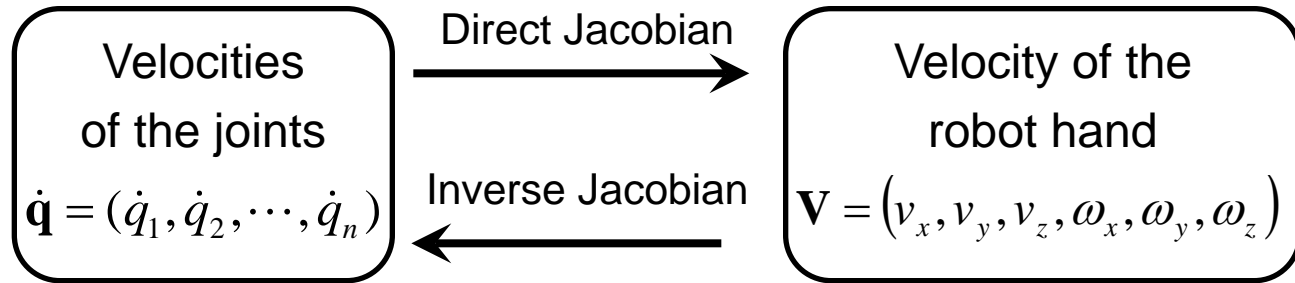
## ■ Inverse geometrical model: $\mathbf{q} = f^{-1}(\mathbf{X})$

- ❑ Motion control (control  $\mathbf{q}$  to obtain  $\mathbf{X}$ ):
- ❑ No systematic method to obtain it and in general no unique solution.
- ❑ System with  $n$  unknowns and 12 possible nonlinear equations (at most 6 independent unknowns).



# Robotic manipulation: Kinematic model

## ■ Differential model



## ■ Direct Jacobian:

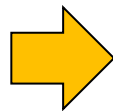
- Hand velocities as a function of joint velocities.

$$\mathbf{V} = \mathbf{J}(\mathbf{q}) \cdot \dot{\mathbf{q}}$$

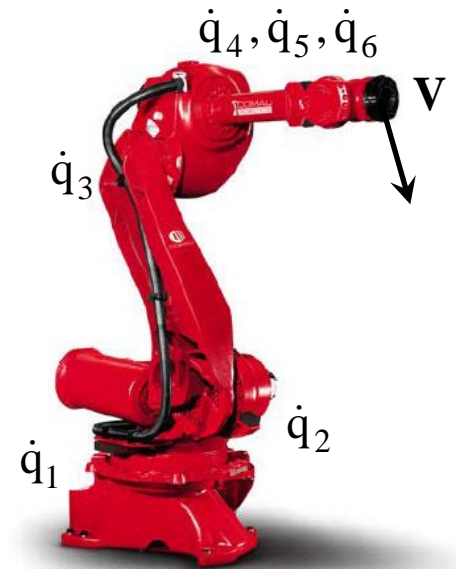
## ■ Jacobian matrix

$$\mathbf{X} = f(\mathbf{q})$$

$$\mathbf{J}(\mathbf{q}) \equiv \frac{\partial f(\mathbf{q})}{\partial \mathbf{q}}$$



$$\begin{pmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \mathbf{J}(\mathbf{q}) \cdot \dot{\mathbf{q}} = \begin{bmatrix} \frac{\partial f_x}{\partial q_1} & \dots & \frac{\partial f_x}{\partial q_n} \\ \frac{\partial f_y}{\partial q_1} & \dots & \frac{\partial f_y}{\partial q_n} \\ \frac{\partial f_z}{\partial q_1} & \dots & \frac{\partial f_z}{\partial q_n} \\ \frac{\partial \omega_x}{\partial q_1} & \dots & \frac{\partial \omega_x}{\partial q_n} \\ \frac{\partial \omega_y}{\partial q_1} & \dots & \frac{\partial \omega_y}{\partial q_n} \\ \frac{\partial \omega_z}{\partial q_1} & \dots & \frac{\partial \omega_z}{\partial q_n} \end{bmatrix} \cdot \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{pmatrix}$$



## ■ Inverse Jacobian: Joint velocities as a function of hand velocities.

$$\mathbf{V} = \mathbf{J}(\mathbf{q}) \cdot \dot{\mathbf{q}} \Rightarrow \dot{\mathbf{q}} = \mathbf{J}^+(\mathbf{q}) \cdot \mathbf{V}$$

# Index

- Introduction
- Robotic manipulation
- **Vision systems**
  - Why vision?
  - Camera model
- Classification of visual control systems
- Position-Based Visual Servoing
- Image-Based Visual Servoing
- Stability analysis
- Hybrid Visual Servoing
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# Vision systems

## ■ Sensors



Ultrasounds



Laser



Odometer



Infrared



Vision  
systems



Contact sensor



3D Laser



IMU

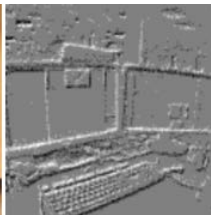
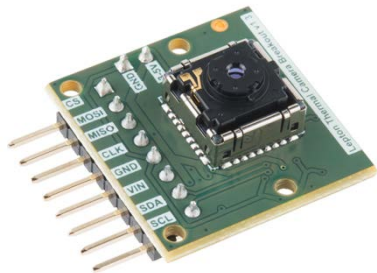


GPS

# Vision systems

## ■ Why vision?

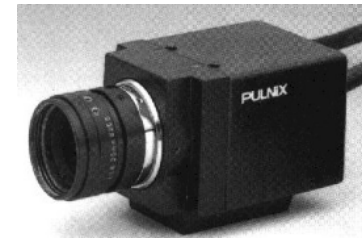
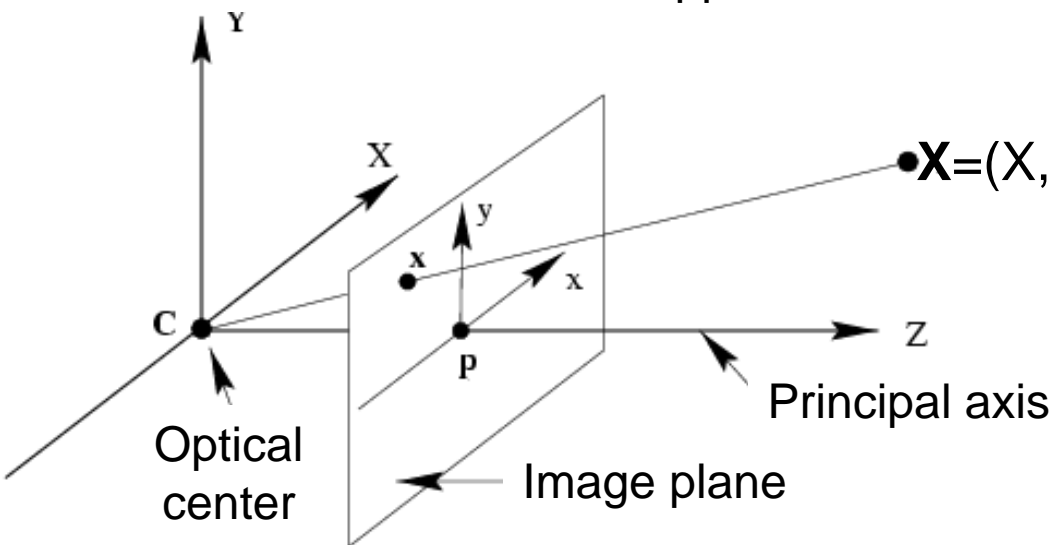
- ❑ Great richness of information
- ❑ Real-time processing
- ❑ Versatile perception system
- ❑ Miniaturization prospects
- ❑ Wide variety of camera types
- ❑ Low cost



# Vision systems: Camera model

## ■ Pinhole camera

- ❑ Captures a set of rays passing through the **center of projection** (center of the camera, focal point).
- ❑ Image is formed in the **image plane**
- ❑ **Pinhole** model as approximation of the actual camera



Intrinsic parameters  
 Perspective Projection  
 Extrinsic parameters

$$\begin{bmatrix} u \\ v \\ s \end{bmatrix} = \lambda \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{CW} & \mathbf{t}_{CW} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

## ■ Perspective projection:

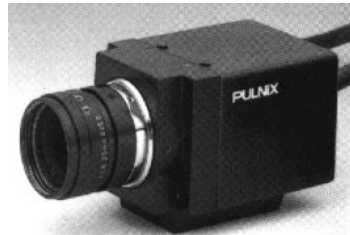
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# Classification of visual servoing systems



**Robot**



**Vision system**



**Control  
law?**



**Visual  
servoing**

# Classification of visual servoing systems

Categories:

- **According to the configuration of the camera and the robot**
  1. Camera in hand
  2. Camera to hand
  3. Multiple cameras
- According to the hierarchical structure of the vision system and the joint control of the robot
  1. Open loop control: perception and action
  2. Systems of dynamic perception and action
  3. Visual direct control
- According to the definition of the error signal
  1. Position-based visual servoing
  2. Image-based visual servoing
  3. Hybrid visual servoing



# Classification of visual servoing systems

## ■ Depending on the camera and robot configuration

### 1. Camera in hand:

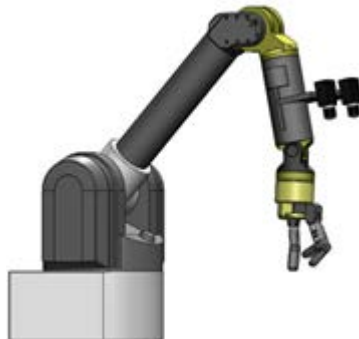
#### ➤ Advantages

- ❖ The camera can observe the movement of the terminal element with fixed resolution and without occlusions.

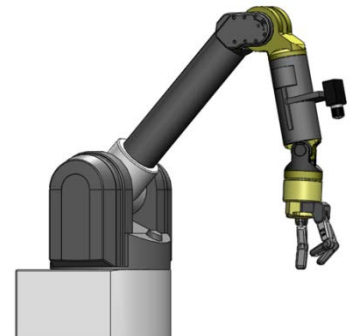
#### ➤ Disadvantages

- ❖ Variable camera position.
- ❖ Field of view can change radically with robot movement.

Stereo  
cameras  
in hand



Camera  
in hand



# Classification of visual servoing systems

## ■ Depending on the camera and robot configuration

### 2. Camera to hand:

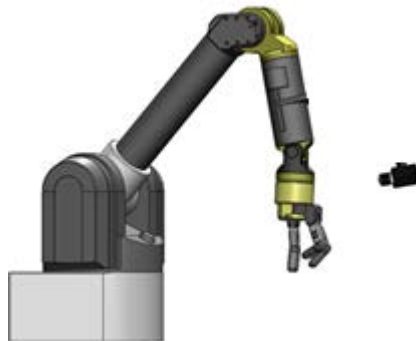
#### ➤ Advantages

- ❖ Field of view remains constant and independent of robot movement.
- ❖ Constant camera position. Offline calibration.

#### ➤ Disadvantages

- ❖ The robot may cause occlusions in the camera field of view during its movement.

Camera  
to hand



Stereo  
cameras  
to hand



# Classification of visual servoing systems

## ■ Depending on the camera and robot configuration

### 3. Multiples cameras:

#### ➤ Advantages

- ❖ Wide combined field of view minimizing occlusions.

#### ➤ Disadvantages

- ❖ Additional processing cost
- ❖ Calibration of multiple cameras
- ❖ Synchronization of cameras

Multiples  
cameras



# Classification of visual servoing systems

Categories:

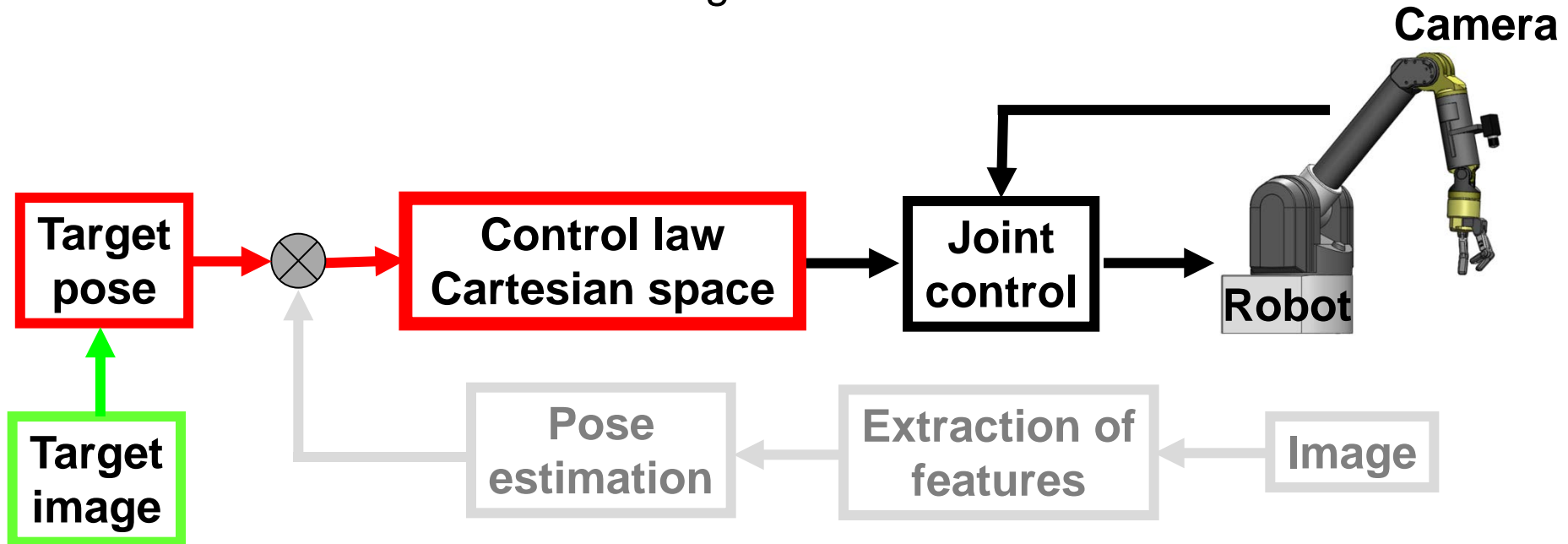
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  3. Hybrid visual servoing

# Classification of visual servoing systems

- According to the hierarchical structure of the vision system and the joint control of the robot

## 1. Open loop control: perception and action

- The extraction of information from the image and control of the robot are two sequential tasks
- The robot executes the task assuming that the working environment is unchanged.

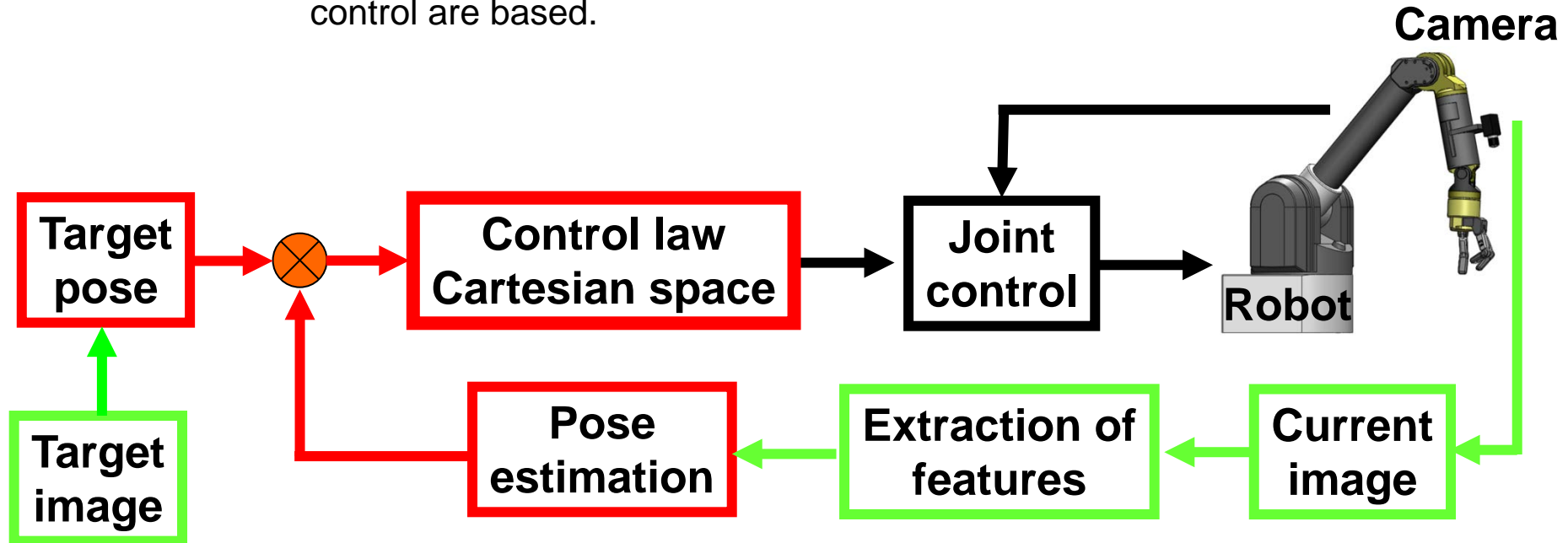


# Classification of visual servoing systems

- According to the hierarchical structure of the vision system and the joint control of the robot

## 2. Systems of dynamic perception and action

- Two control loops at different frequencies:
  - ❖ Internal servomotor control loop at high frequency.
  - ❖ External loop of visual perception at a lower frequency.
  - ❖ This is the category on which most implementations in the field of visual control are based.





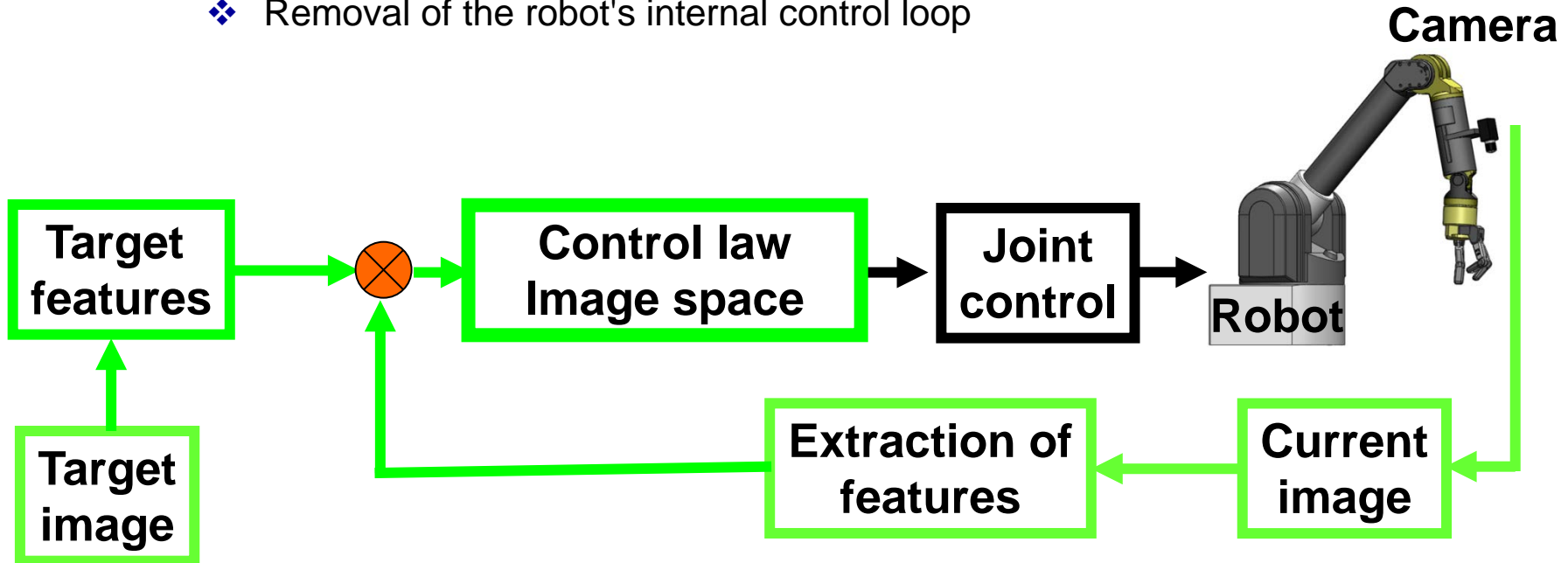
# Classification of visual servoing systems

- According to the hierarchical structure of the vision system and the joint control of the robot

## 3. Visual direct control

### ➤ A control loop

- ❖ Visual perception at high frequency
- ❖ Eliminate the robot controller, replacing it with a controller that only uses vision to stabilize the mechanism
- ❖ Removal of the robot's internal control loop



# Classification of visual servoing systems

- According to the hierarchical structure of the vision system and the joint control of the robot
- Traditionally there has been a clear predominance of *look and move* (perception and action) systems over direct servo control systems:
  - The slow sampling rate available for vision and the use of complex, nonlinear dynamics by direct servo control systems result in a complex control problem.
  - Look-and-move control methods separate the kinematic peculiarities of the mechanism from visual control, whereas for direct servo control there is a strong dynamic coupling.
  - Internal joint control loops with a high sampling rate exhibit good dynamics.
  - Many controllers have specialized mechanisms to deal with kinematic singularities, thus simplifying system design.

# Classification of visual servoing systems

Categories:

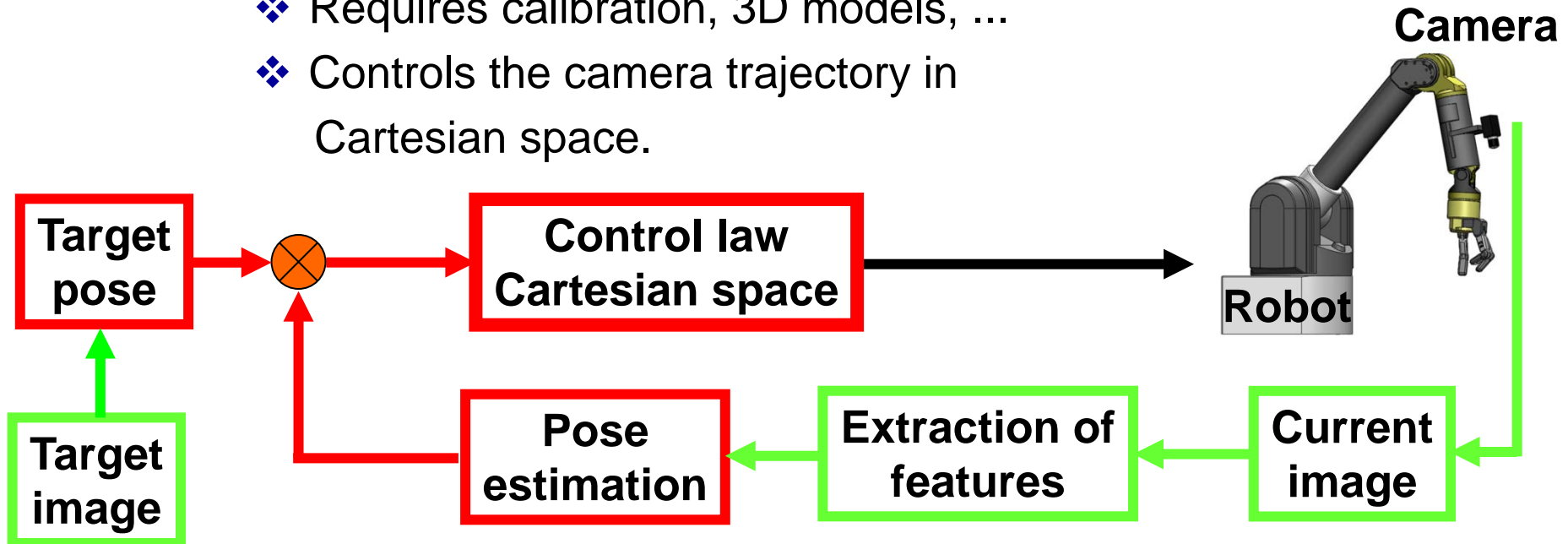
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  3. Hybrid visual servoing

# Classification of visual servoing systems

## ■ According to the definition of the error signal

### 1. Position-based visual servoing

- Estimation of position and orientation of the target
- The task is defined in 3D space
  - ❖ Feedback of current position of the terminal element.
  - ❖ Requires calibration, 3D models, ...
  - ❖ Controls the camera trajectory in Cartesian space.

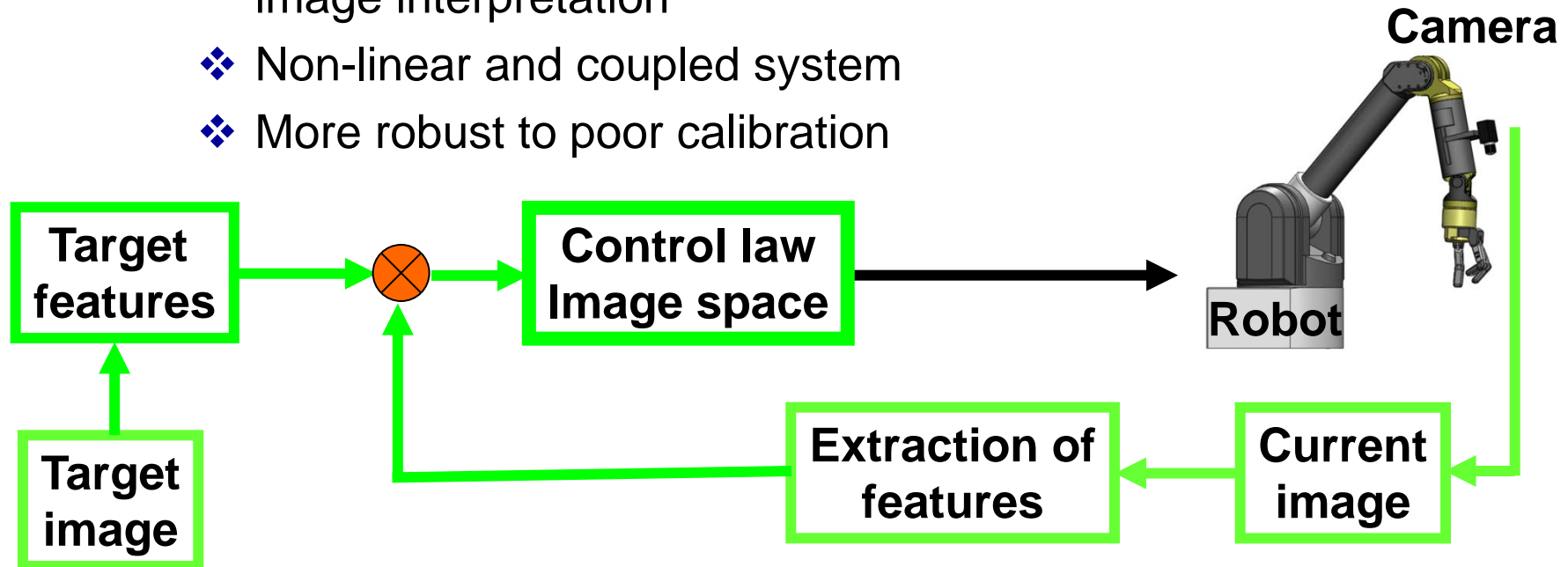


# Classification of visual servoing systems

## ■ According to the definition of the error signal

### 2. Image-based visual servoing

- The target is defined in image coordinates (pixel).
- Task is defined in the image
  - ❖ Can reduce computational delay, eliminating the need for image interpretation
  - ❖ Non-linear and coupled system
  - ❖ More robust to poor calibration

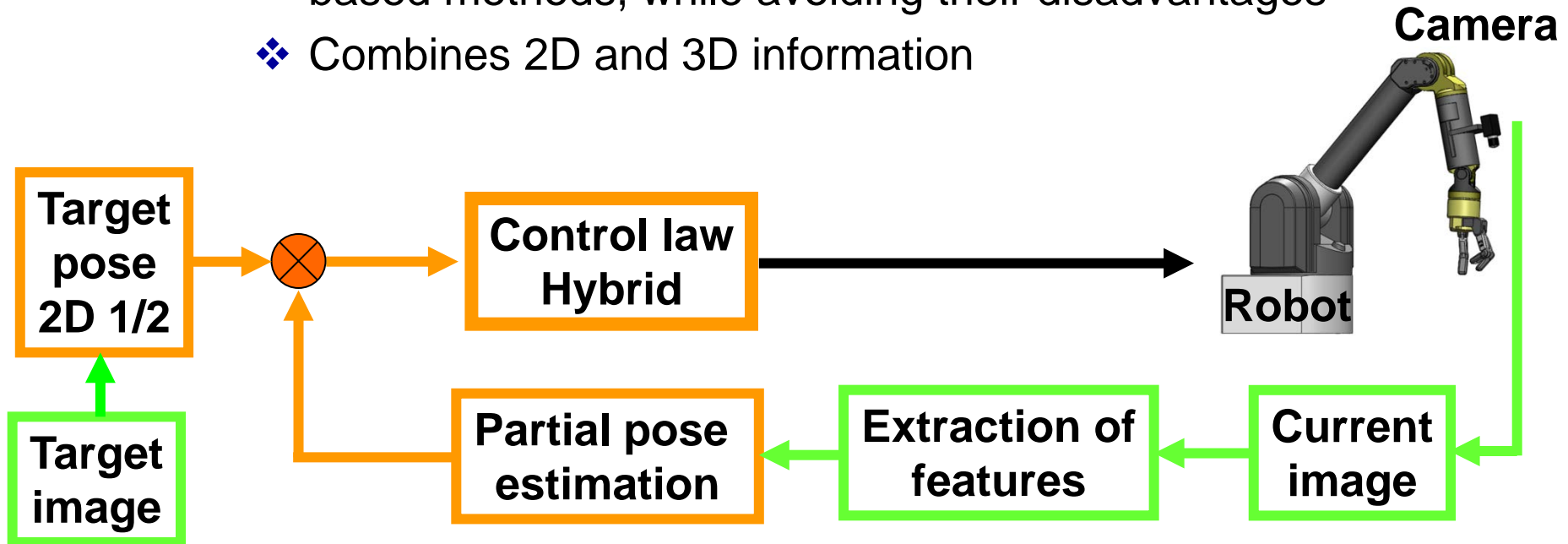


# Classification of visual servoing systems

## ■ According to the definition of the error signal

### 3. Hybrid visual servoing

- Partially position-based and image-based
  - ❖ Partial decoupling in the interaction matrix
  - ❖ Presents the advantages of the position-based and image-based methods, while avoiding their disadvantages
  - ❖ Combines 2D and 3D information





- Introduction
- Robotic manipulation
- Vision systems
- Classification of visual control systems
  
- **Position-Based Visual Servoing**
  - Based on model
  - Based on epipolar geometry
  - Based on homography
  - Control law
  
- Image-Based Visual Servoing
- Stability analysis
- Hybrid Visual Servoing
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# Position-Based Visual Servoing

## ■ Position-based visual servoing

- Vision data is used to build a 3D representation of the environment.

- The 3D position of the manipulator relative to its desired position is estimated from the acquired images.

- ❖ Based on 3D model

- ❖ Two-view geometry

- ❖ Stereo view

- Control law that eliminates the error in 3D space.

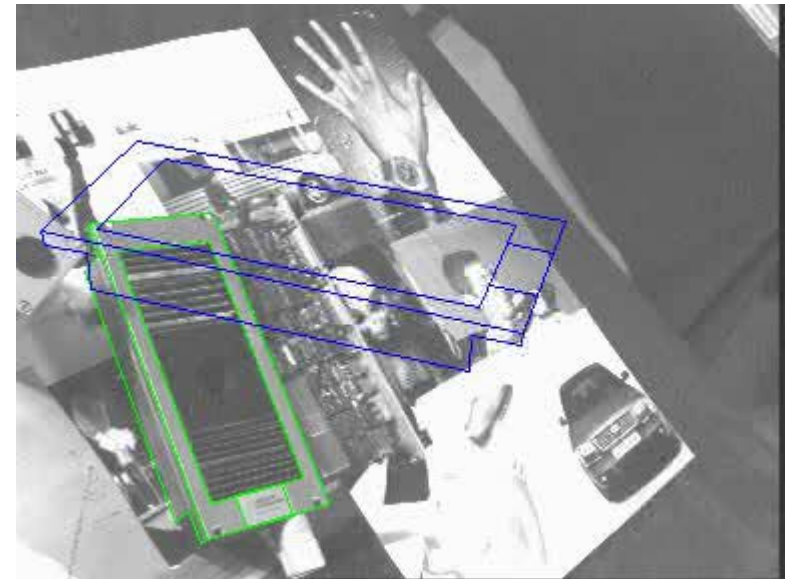
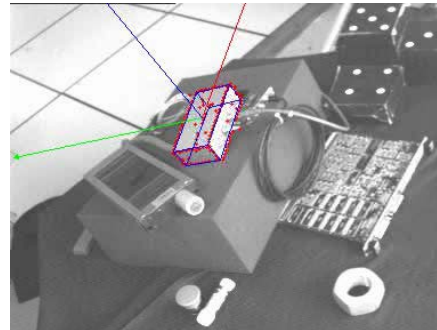
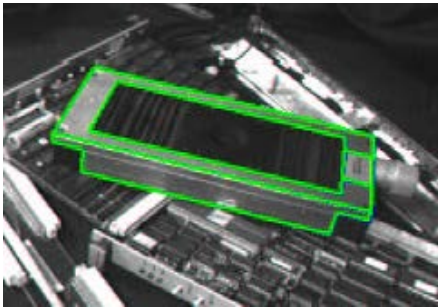
- Related bibliography :

- [Taylor et al. 1985], [Zhang et al. 1990], [Carlsson 1991], [Tonko et al. 1997], [Basri et al. 1998], [Martinet & Gallice 1999], [Taylor & Ostrowski 2000], [Liang & Pears 2002], [Benhimane et al. 2005].

# Position-Based Visual Servoing: Model

## ■ 3D localization estimation: Model based

- The objective of the control is to move the robot camera until the 3D model projection of the object corresponds to the observed image.



- It is required to know
  - 3D model of the object (CAD)
  - Intrinsic camera calibration
  - Matching of features in the 3D model with features of its projection on the image
- Dementhon's algorithm for estimating object location

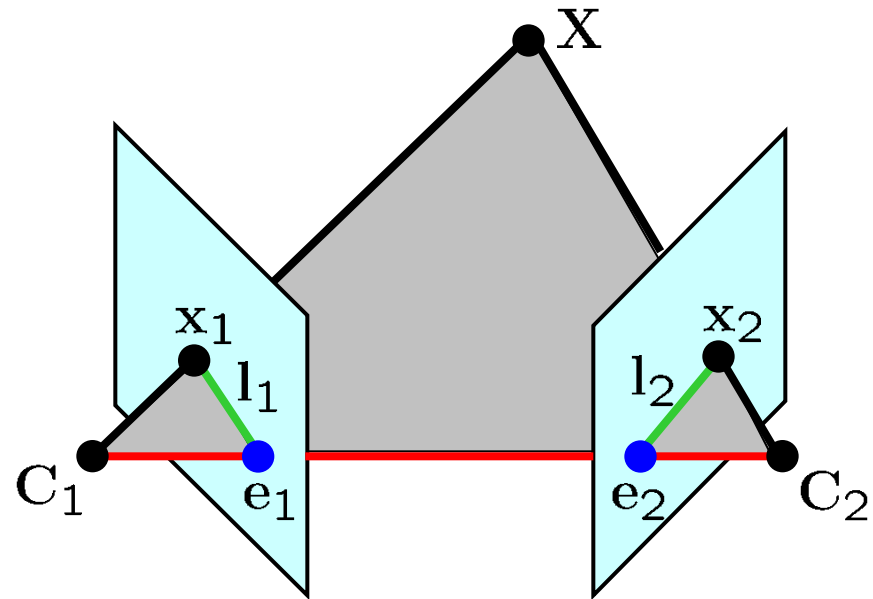
# Position-Based Visual Servoing: Epipolar geometry

## ■ 3D pose estimation: Epipolar Geometry

- Intrinsic geometry between two views. It depends only on the internal parameters of the cameras and their relative position.
- The fundamental matrix encapsulates this intrinsic geometry.

$$l_2 = F x_1$$

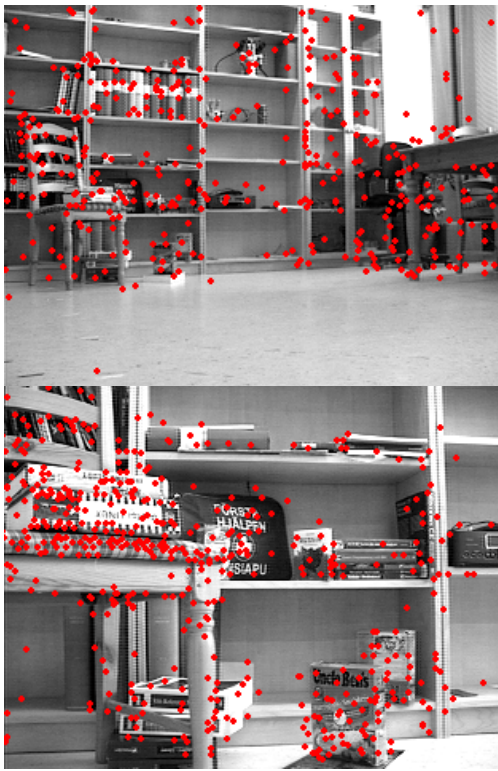
$$x_2^T F x_1 = 0$$



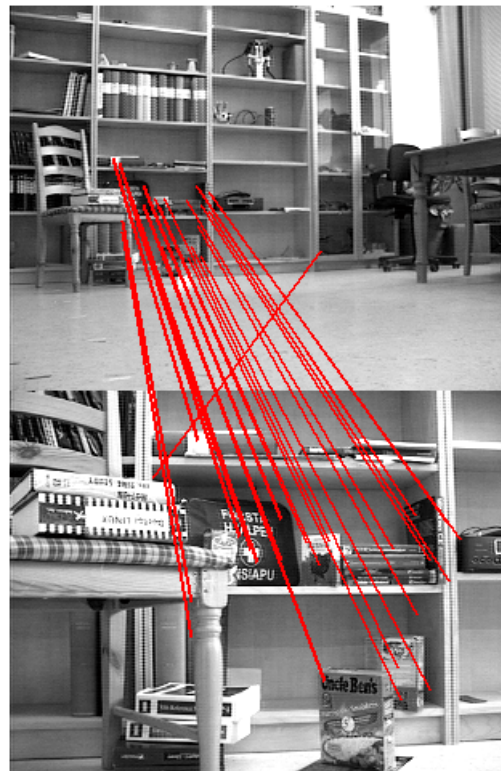
- **Baseline**: Line joining the centers of the cameras.
- **Epipole**: Intersection between the baseline and the image.
- **Epipolar plane**: Plane containing the baseline.
- **Epipolar line**: Intersection of an epipolar plane with the image.

# Position-Based Visual Servoing: Epipolar geometry

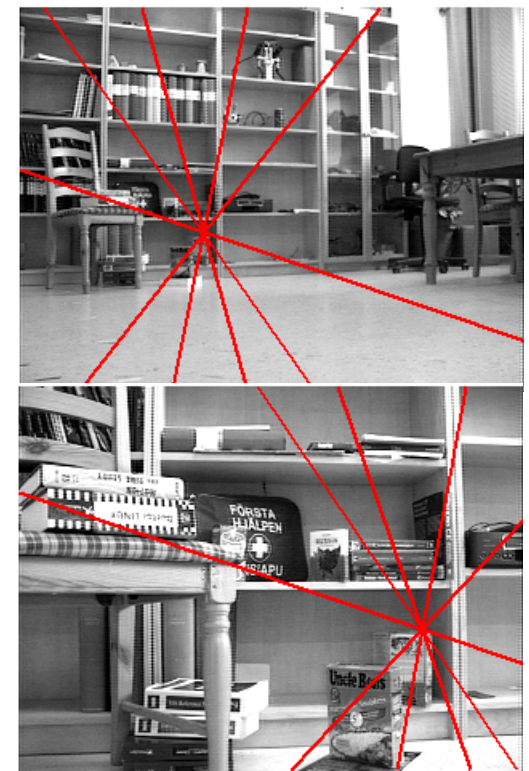
- Epipolar geometry estimation
  - Automatic image feature extraction and matching (SIFT, Harris points,...)
  - Robust fundamental matrix estimation (RANSAC, MAPSAC,...)



Feature extraction



Point matching



Epipolar geometry estimation

# Position-Based Visual Servoing: Epipolar geometry

- Essential Matrix = Calibrated Fundamental Matrix

$$\mathbf{P}_1 = [\mathbf{I}; \mathbf{0}]; \quad \mathbf{P}_2 = [\mathbf{R}_{21}; \mathbf{t}_{21}] \Rightarrow \mathbf{x}_2^T \mathbf{E}_{21} \mathbf{x}_1 = 0$$

$$\mathbf{E}_{21} = [\mathbf{t}_{21}]_{\times} \mathbf{R}_{21}$$

$$\text{since } \mathbf{F}_{21} = \mathbf{K}_2^{-T} [\mathbf{t}_{21}]_{\times} \mathbf{R}_{21} \mathbf{K}_1^{-1} \Rightarrow \mathbf{E}_{21} = \mathbf{K}_2^T \mathbf{F}_{21} \mathbf{K}_1$$

$$\mathbf{p}_2^T \mathbf{F}_{21} \mathbf{p}_1 = 0 \quad \text{with} \quad \mathbf{p}_1 = \mathbf{K}_1 \mathbf{x}_1; \quad \mathbf{p}_2 = \mathbf{K}_2 \mathbf{x}_2$$

- Decomposition of the essential matrix to obtain rotation and translation (up to scale factor).
- Algorithm:

$$\mathbf{E} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^T$$

$$\mathbf{t} = \mathbf{U} \cdot (0,0,1)^T$$

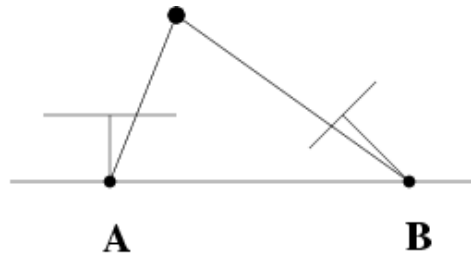
$$\mathbf{R}_1 = \mathbf{U} \cdot \mathbf{W} \cdot \mathbf{V}^T$$

$$\mathbf{R}_2 = \mathbf{U} \cdot \mathbf{W}^T \cdot \mathbf{V}^T$$

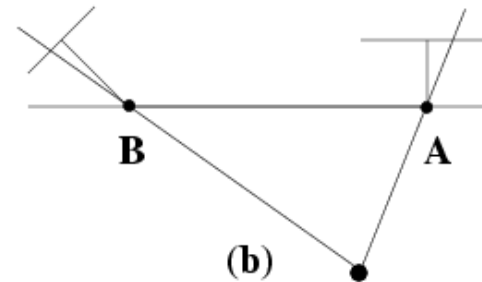
$$\text{With } \mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Position-Based Visual Servoing: Epipolar geometry

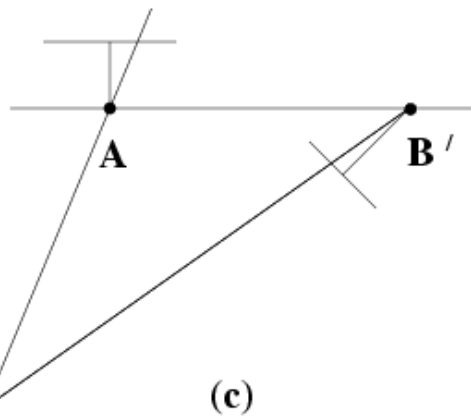
- The essential matrix decomposition yields four theoretical solutions:  $(\mathbf{t}, \mathbf{R}_1), (\mathbf{t}, \mathbf{R}_2), (-\mathbf{t}, \mathbf{R}_1), (-\mathbf{t}, \mathbf{R}_2)$



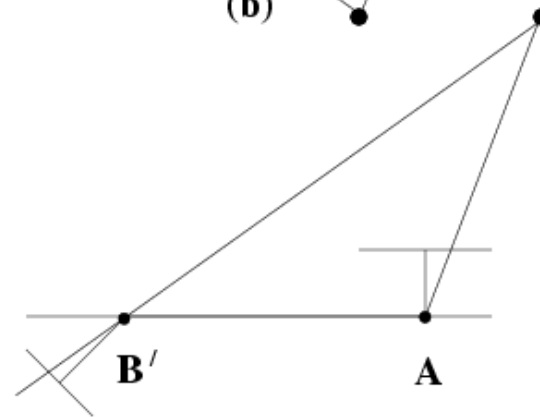
(a)



(b)



(c)



(d)



# Position-Based Visual Servoing: Homography

## ■ Homography:

- Two images can be geometrically related by a homography induced by a plane of the scene:

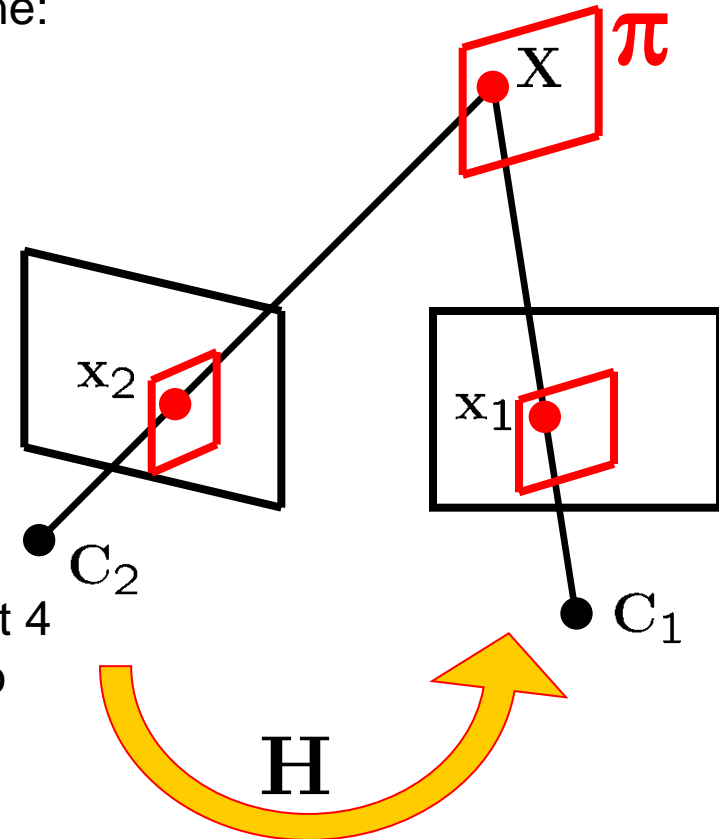
$$\mathbf{H} \in \mathbb{R}^{3 \times 3}$$

- Corresponding points related by a homography:

$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1$$

- Homography can be computed from at least 4 corresponding points of the plane in the two images.

$$\mathbf{H} = \mathbf{K}_2 \left( \mathbf{R} - \mathbf{t} \mathbf{n}^T / d \right) \mathbf{K}_1^{-1}$$



# Position-Based Visual Servoing: Homography

## ■ Homography decomposition:

$$H_c = K^{-1} \cdot H \cdot K$$

$$[U, S, V] := \text{svd}(H_c)$$

$$H_c = H_c / S_{22}$$

$$V := [v_1, v_2, v_3]$$

$$(S)^2 := \text{diag}[\lambda_1, \lambda_2, \lambda_3]$$

$$\alpha = \sqrt{\frac{\lambda_3 - \lambda_2}{\lambda_3 - \lambda_1}}, \beta = \sqrt{\frac{\lambda_2 - \lambda_1}{\lambda_3 - \lambda_1}}$$

$$\omega_1 = \alpha \cdot v_1 + \beta \cdot v_3$$

$$\omega_2 = \alpha \cdot v_1 - \beta \cdot v_3$$

$$U_1 = [\omega_1, v_2, (\omega_1 \times v_2)]$$

$$U_2 = [\omega_2, v_2, (\omega_2 \times v_2)]$$

$$W_1 = [H_c \cdot \omega_1, H_c \cdot v_2, (H_c \cdot \omega_1) \times (H_c \cdot v_2)]$$

$$W_2 = [H_c \cdot \omega_2, H_c \cdot v_2, (H_c \cdot \omega_2) \times (H_c \cdot v_2)]$$

Solutions  $i$  ( $i = 1, 2$ ):

$$R_i = W_i \cdot U_i^T$$

$$n_i = \omega_i \times v_2$$

$$t_i = (I_{3 \times 3} - R_i^T \cdot H_c) \cdot n_i \cdot d$$

$d$ : Distance to the plane

# Position-Based Visual Servoing: Control law

## ■ Control law

- Given the location of the terminal element:  ${}^W\mathbf{T}_H(\mathbf{x}_H)$
- The goal is to reach location:  ${}^W\mathbf{T}_{Ht}(\mathbf{x}_{Ht})$
- The task of matching the terminal element to a desired location can be defined as.

$$\varepsilon(\mathbf{x}_H(t)) = \mathbf{x}_{Ht} - \mathbf{x}_H(t)$$

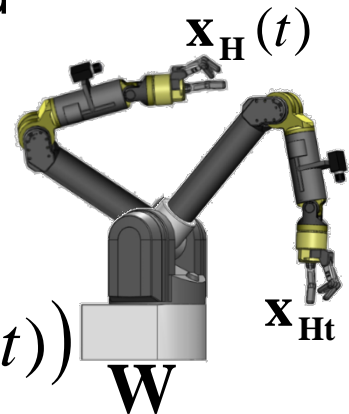
- The simplest control law that assures zero error with exponential decay is

$$\dot{\mathbf{x}}_H(t) = K \cdot \varepsilon(\mathbf{x}_H(t)) = K \cdot (\mathbf{x}_{Ht} - \mathbf{x}_H(t))$$

- Controller gain  $K$
- The output of the controller is the desired speed of the terminal element to reach the target.
- The inverse Jacobian allows to calculate the joint velocities required to perform the task:

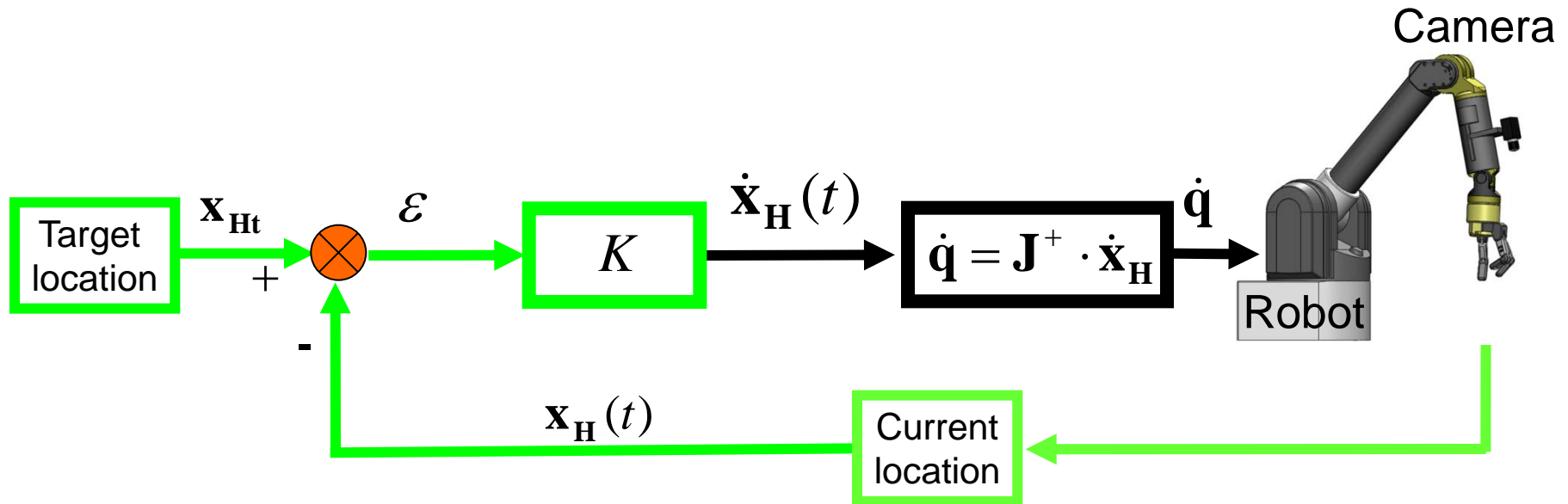
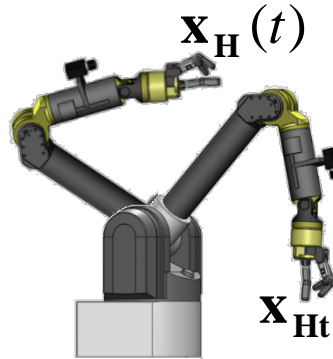
$$\dot{\mathbf{q}} = \mathbf{J}^+(\mathbf{q}) \cdot \dot{\mathbf{x}}_H(t)$$

$${}^W\mathbf{T}_H \in \mathcal{R}^{4 \times 4}$$
$$\mathbf{x}_H \in \mathcal{R}^6$$



# Position-Based Visual Servoing: Control law

## ■ Control law



# Position-Based Visual Servoing: Control law

## ■ Control law for tracking $\mathbf{x}_{Ht}(t)$

□ If the desired configuration varies over time the error will not converge to zero.

□ Use proportional and integral control (PI).

➤ Estimate the error variation at instant k:  $\mathbf{I}_k$

$$\mathbf{I}_{k+1} = \mathbf{I}_k + \mu \cdot \mathbf{e}_k = \mu \cdot \sum_{j=0}^k \mathbf{e}_j \quad \text{with} \quad \mathbf{I}_0 = 0$$

➤ Efficient for tracking a target at constant speed.

$$\mathbf{I}_{k+1} = \mathbf{I}_k \quad \text{if} \quad \mathbf{e}_k = 0$$

## ■ Conclusions

- ❑ It needs a priori information (calibration). Therefore unsuitable for unstructured environments.
- ❑ Can be unstable if the initial position of the camera is far away from the desired position.
- ❑ Directly controls the camera trajectory in Cartesian space. Straight trajectory in 3D space.
- ❑ Decoupled control and no singular configurations.
- ❑ No control in the image. Points can leave the field of view:

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