Assistive Robotics

Hybrid Visual Servoing and Stability Analysis

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Index

- Introduction
- Robotic manipulation
- Vision systems
- Classification of visual control systems
- Position-Based Visual Servoing
- Image-Based Visual Servoing

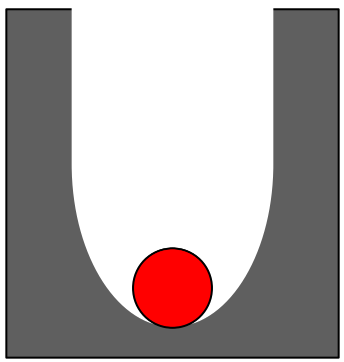
- **Stability analysis**
  - Lyapunov stability
  - Image-based visual servoing stability

- Hybrid Visual Servoing
- Conclusion and bibliography

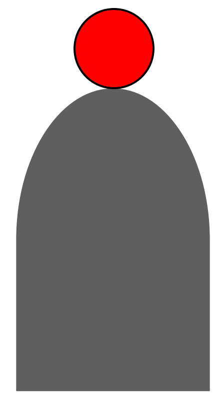
Stability analysis

- It is important that the systems are stable. An unstable system is generally useless and potentially dangerous.
- A system is said to be stable if the system being close to its operating point implies that it will always remain around that point.

Stable equilibrium point



Unstable equilibrium point



- Stability of systems
  - Time invariant linear systems:
    - Multiple techniques and criteria available (Nyquist, Routh, etc.).
  - Non-linear or time-varying systems:
    - Difficult and sometimes impossible
  - Lyapunov method

Stability analysis: Lyapunov

- Lyapunov's methods (1892)
  - For determining the stability of dynamical systems described by ordinary differential equations.
  - First Lyapunov method
    - Applicable when the explicit solution of the differential equations is available.
  - Lyapunov's second method or direct method
    - To analyze the stability without solving the differential equations.

Aleksandr Liapunov
1857-1918
Russian mathematician and physicist

- The basic idea of Lyapunov's direct method is based on the physical fact that if the total "energy" of a mechanical system is continuously dissipated, then, whether the system is linear or nonlinear, it must eventually reach a point of equilibrium (zero energy).
- Therefore, the stability of the system can be analyzed by studying the variation of an energy function associated with the system.

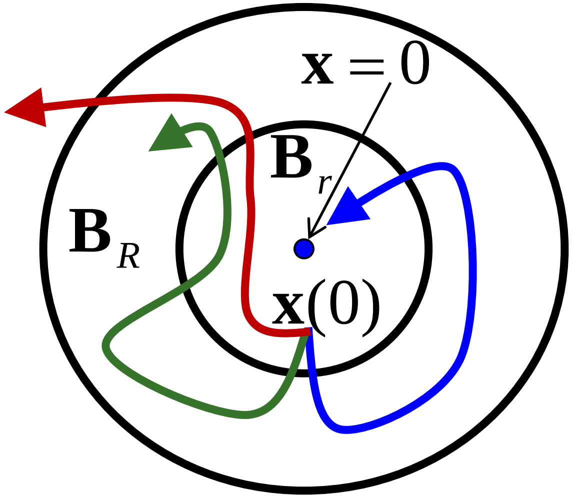
Stability analysis: Lyapunov

- Lyapunov stability
  - Let be a system with state vector $\mathbf{x}$ such that: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$
  - We define state or equilibrium point of the system: $\mathbf{f}(\mathbf{x_e} = \mathbf{0}, t) = \mathbf{0}, \; \forall t$
    - If the system is linear: $\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x}, \;$ if $\det(\mathbf{A}) \neq 0 \Rightarrow \exists$ one single $\mathbf{x_e}$
    - In nonlinear systems there may be more than one equilibrium point.
  - Lyapunov stability:

    $$\forall R > 0, \; \exists r > 0, \; \mathbf{x}(0) \in \mathbf{B}_r \Rightarrow \forall t \geq 0, \; \mathbf{x}(t) \in \mathbf{B}_R$$

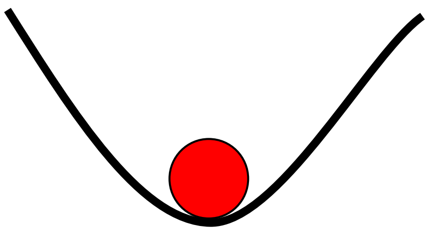
    - An equilibrium point is **stable** if for any R>0 there exists an r>0 such that if $\mathbf{x}(0)$ belongs to the sphere of radius r, then $\mathbf{x}(t)$ will belong to the sphere of radius R for all t.
    - Otherwise the equilibrium point is **unstable**.
  - Asymptotic stability:
    - An equilibrium point $\mathbf{x}=\mathbf{0}$ is **asymptotically stable** if it is stable and in addition

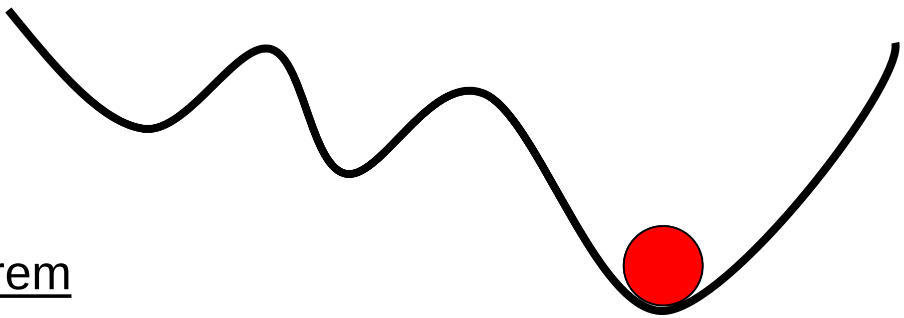
    $$\exists r > 0, \mathbf{x}(0) \in \mathbf{B}_r \Rightarrow \mathbf{x}(t) \rightarrow 0, t \rightarrow \infty$$



Stability analysis: Lyapunov

- Lyapunov stability
  - Global: If asymptotic stability is satisfied for any initial state, the equilibrium point is globally asymptotically stable.
  - Local: If asymptotic stability is not satisfied for every initial state, the equilibrium point is locally asymptotically stable.





  - <u>Lyapunov's direct method theorem</u>

$$\text{Given the system } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t), \text{ with } \mathbf{f}(\mathbf{0}, t) = \mathbf{0}, \ \forall t, \ \text{if } \exists \mathrm{V}(\mathbf{x}, t) \ni$$

$$\mathrm{V}(\mathbf{x}, t) > 0 \text{ (positive definite) and } \dot{\mathrm{V}}(\mathbf{x}, t) < 0 \text{ (negative definite)}$$

$$\Rightarrow \text{The equilibrium point in the origin is asymptotically stable}$$

  - The scalar function $\mathbf{V}(\mathbf{x}, \mathrm{t})$ is called Lyapunov function
  - A function $\mathbf{V}$ is positive definite if $\mathrm{V}(\mathbf{x}) > 0, \forall \mathbf{x} \neq \mathbf{0} \wedge \mathrm{V}(\mathbf{0}) = 0$

Stability analysis: Lyapunov

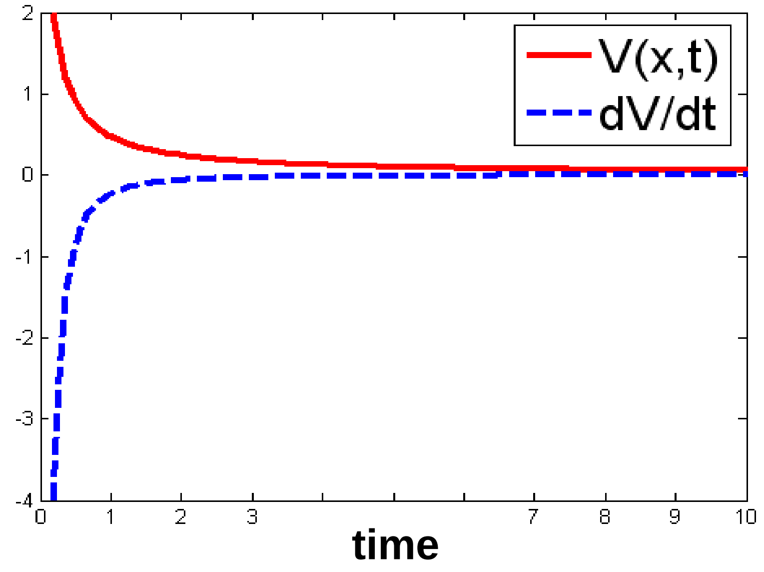
- Example of Lyapunov stability

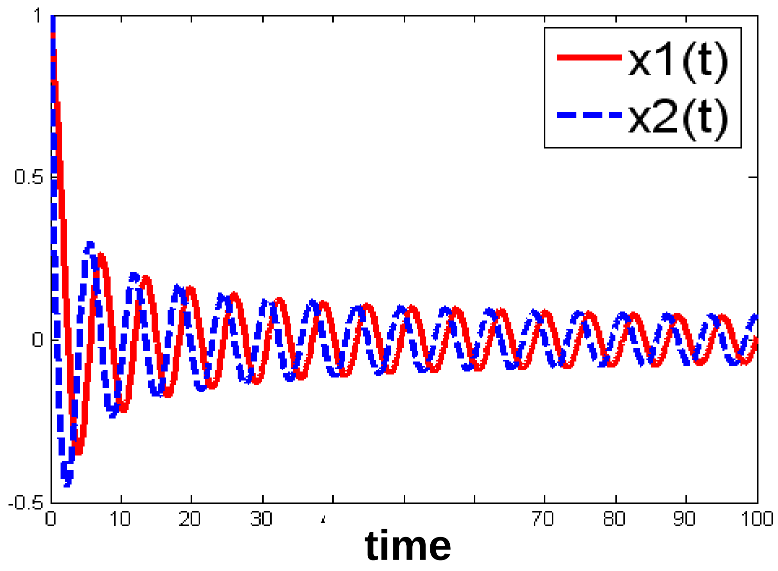
$$\begin{cases} \dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2) \\ \dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2) \end{cases}$$

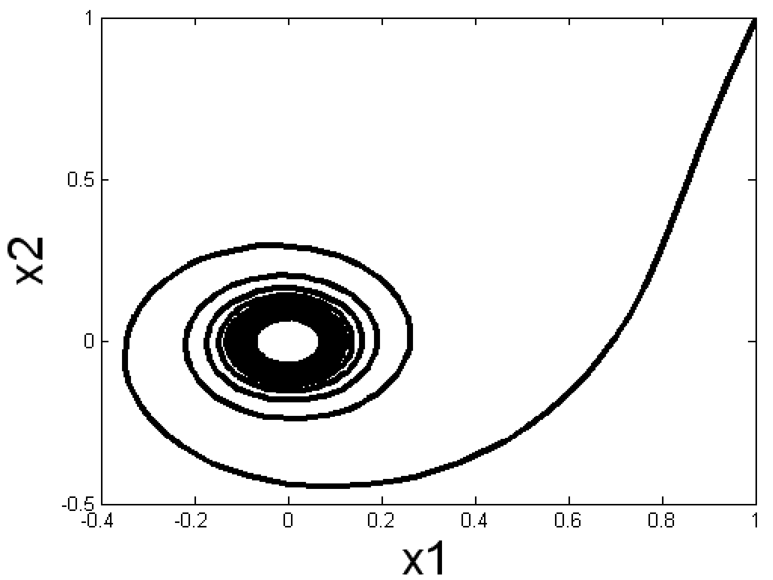
$$V(\mathbf{x}) = x_1^2 + x_2^2 > 0$$

$$\dot{V}(\mathbf{x}) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2$$

$$= -2(x_1^2 + x_2^2)^2 < 0$$







Stability analysis: Visual servoing

- Lyapunov stability for image-based visual servoing
  - Control law: $\dot{\mathbf{e}} = \mathbf{L} \cdot \mathbf{v}$, with $\mathbf{v} = -\lambda \cdot \hat{\mathbf{L}}^{+} \cdot \mathbf{e}$
  - We define the Lyapunov function: $\mathrm{V}(\mathbf{e}) = \frac{1}{2} \cdot \left\| \mathbf{e}(\mathrm{t}) \right\|^{2} > 0$
  - Derivate to obtain the stability condition:

$$\dot{\mathrm{V}} = \mathbf{e}^{\mathbf{T}} \cdot \dot{\mathbf{e}} = -\lambda \cdot \mathbf{e}^{\mathbf{T}} \cdot \mathbf{L} \cdot \hat{\mathbf{L}}^{+} \cdot \mathbf{e} < 0$$

    - Which is satisfied if the matrix $(\mathbf{L} \cdot \hat{\mathbf{L}}^{+})$ is positive definite:

$$\mathbf{L} \cdot \hat{\mathbf{L}}^{+} > 0 \Rightarrow \text{Asymptotically stable}$$

    - Note: A symmetric matrix is positive definite if for every non-zero vector its quadratic form is positive

$$\mathbf{A} \in \Re^{n \times n}, \ \mathbf{A} = \mathbf{A}^{\mathbf{T}}$$

$$\mathbf{A} > 0 \Leftrightarrow \forall \mathbf{b} \neq \mathbf{0}, \ \mathbf{b}^{\mathrm{T}} \cdot \mathbf{A} \cdot \mathbf{b} > 0$$

$$\text{e.g. } \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \mathbf{A} > 0 \Leftrightarrow |a_{11}| > 0, \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} > 0$$

Stability analysis: Visual servoing

$$\mathbf{L}\cdot\hat{\mathbf{L}}^{+} > 0 \Rightarrow \text{Stable} \qquad \mathbf{L} \in \Re^{k\times m}$$

- m = number of degrees of freedom (in general m=6)
- k = number of features (e.g. k=2n with n points)
- Assuming maximum rank

$k = m, rank(\mathbf{L}) = m$ | $k < m, rank(\mathbf{L}) = k$ | $k > m, rank(\mathbf{L}) = m$

- Number of features equal to the number of degrees of freedom of the camera

$$\left(\mathbf{L}\cdot\hat{\mathbf{L}}^{+}\right) \in \Re^{m\times m} \rightarrow \mathbf{L}\cdot\hat{\mathbf{L}}^{+} > 0$$

- Ideally one would have:

$$\mathbf{L}\cdot\hat{\mathbf{L}}^{+} = \mathbf{L}\cdot\hat{\mathbf{L}}^{-1} = \mathbf{I} > 0$$

- The estimation is assumed to be reasonably accurate

Stability analysis: Visual servoing

$$\mathbf{L} \cdot \hat{\mathbf{L}}^{+} > 0 \Rightarrow \text{Stable} \qquad \mathbf{L} \in \Re^{k \times m}$$

- m = number of degrees of freedom (in general m=6)
- k = number of features (e.g. k=2n with n points)
- Assuming maximum rank

$k = m, rank(\mathbf{L}) = m$	$k < m, rank(\mathbf{L}) = k$	$k > m, rank(\mathbf{L}) = m$

- Number of features less than the degrees of freedom of the camera.
  - For example, for a point (k=2) and a camera with 6 degrees of freedom (m=6):

$$\mathbf{L} \in \Re^{2 \times 6} \rightarrow \mathbf{L}^{+} = \mathbf{L}^{T} \cdot \left(\mathbf{L} \cdot \mathbf{L}^{T}\right)^{-1} \rightarrow \left(\mathbf{L} \cdot \hat{\mathbf{L}}^{+}\right) \in \Re^{2 \times 2}$$

- The null space (kernel) of the interaction matrix is defined:

$$\ker(\mathbf{L}) = \{\mathbf{v} \in \Re^{6} : \mathbf{L} \cdot \mathbf{v} = \mathbf{0}\}$$

  - The kernel is a subspace of dimension (m-k). Example: 6-2=4
- Not all camera movements will be observable

$$\mathbf{v} \in \ker(\mathbf{L})$$

  - For example, movement of the camera along the projection ray of the point, or rotation of the camera along the projection ray of the point
- Hybrid schemes: Some degrees of freedom are controlled with image-based control and the rest with other techniques

Stability analysis: Visual servoing

$$\mathbf{L}\cdot\hat{\mathbf{L}}^{+} > 0 \Rightarrow \text{Stable} \qquad \mathbf{L}\in\Re^{k\times m}$$

- m = number of degrees of freedom (in general m=6)
- k = number of features (e.g. k=2n with n points)
- Assuming maximum rank

$k = m, rank(\mathbf{L}) = m$

$k < m, rank(\mathbf{L}) = k$

$k > m, rank(\mathbf{L}) = m$

- Number of features greater than the camera's degrees of freedom

$$\mathbf{L}\in\Re^{k\times m} \rightarrow \mathbf{L}^{+} = \left(\mathbf{L}^{T}\cdot\mathbf{L}\right)^{-1}\cdot\mathbf{L}^{T} \rightarrow \left(\mathbf{L}\cdot\hat{\mathbf{L}}^{+}\right)\in\Re^{k\times k}$$

- The following null kernel is defined

$$\ker(\mathbf{L}^{+}) = \{\mathbf{e}\in\Re^{k} : \mathbf{L}^{+}\cdot\mathbf{e} = \mathbf{0}\}$$

  - The kernel is a subspace of dimension (k-m)
- There are configurations of points, different than the target, that produce zero velocity: $\mathbf{e}\in\ker(\mathbf{L}^{+})$
- The system converges exponentially to a solution different than the desired one: **local minimum**
- Therefore only asymptotic local stability can be demonstrated.
  - Example: k-m=8-6=2. The null space has dimension 2. However this does not necessarily imply that there are 2 local minima, since they must be physically coherent.

Stability analysis: Visual servoing

$$\mathbf{L}\cdot\hat{\mathbf{L}}^{+} > 0 \Rightarrow \text{Stable} \qquad \mathbf{L}\in\Re^{k\times m}$$

- m = number of degrees of freedom (in general m=6)
- k = number of features (e.g. k=2n with n points)
- Assuming maximum rank

$$k = m,\ rank(\mathbf{L}) = m \qquad k < m,\ rank(\mathbf{L}) = k \qquad k > m,\ rank(\mathbf{L}) = m$$

- With k>m the stability condition cannot be fulfilled.

$$k > m \Rightarrow (\mathbf{L}\cdot\hat{\mathbf{L}}^{+}) \in \Re^{k\times k} \Rightarrow rank(\mathbf{L}\cdot\hat{\mathbf{L}}^{+}) \le m \Rightarrow \mathbf{L}\cdot\hat{\mathbf{L}}^{+} \le 0$$

- Asymptotic local stability
- We define a new error: $\mathbf{e}' = \hat{\mathbf{L}}^{+}\cdot\mathbf{e}$
- Derivative of the error:

$$\dot{\mathbf{e}}' = \dot{\hat{\mathbf{L}}}^{+}\cdot\mathbf{e} + \hat{\mathbf{L}}^{+}\cdot\dot{\mathbf{e}} = \hat{\mathbf{L}}^{+}\cdot\mathbf{L}\cdot\mathbf{v} + \mathbf{O}\cdot\mathbf{v} = (\hat{\mathbf{L}}^{+}\cdot\mathbf{L} + \mathbf{O})\cdot\mathbf{v}$$

$$\text{with } \mathbf{O}\in\Re^{6\times 6} \ni \mathbf{e}\to\mathbf{0} \Rightarrow \mathbf{O}\to\mathbf{0}$$

- Closed-loop behavior with the control law

$$\dot{\mathbf{e}}' = -\lambda\cdot(\hat{\mathbf{L}}^{+}\cdot\mathbf{L} + \mathbf{O})\cdot\hat{\mathbf{L}}^{+}\cdot\mathbf{e} = -\lambda\cdot(\hat{\mathbf{L}}^{+}\cdot\mathbf{L} + \mathbf{O})\cdot\mathbf{e}'$$

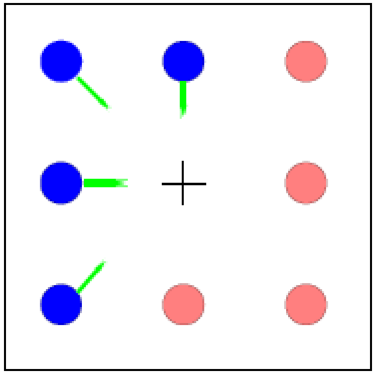
- It is locally asymptotically stable if

$$\hat{\mathbf{L}}^{+}\cdot\mathbf{L} > 0 \quad \text{around} \quad \mathbf{e} = \mathbf{0}$$

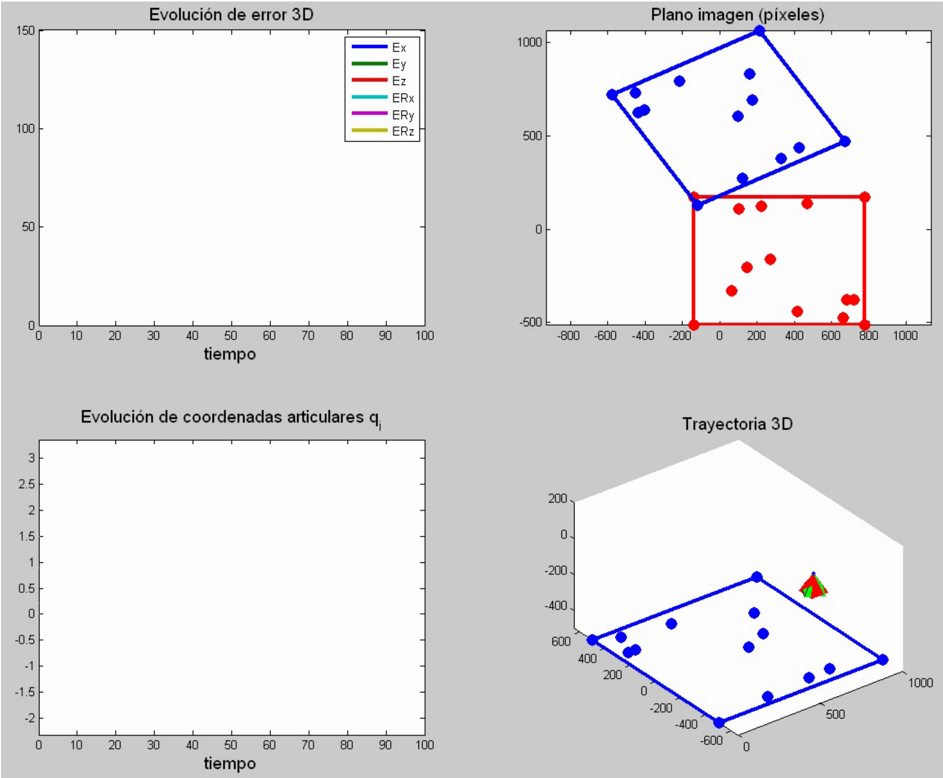
- Provided the estimation is reasonably accurate, this condition will be satisfied if $rank(\mathbf{L}) = rank(\hat{\mathbf{L}}^{+}) = 6$

Stability analysis: Visual servoing

- Singularity issues $\hat{\mathbf{L}} \rightarrow \hat{\mathbf{L}}^{+}$
  - The interaction matrix can be singular:
    - If it is defined by three collinear points
    - If the points and the optical center belong to a cylinder
    - The use of more points reduces the problem but does not guarantee non-singularity



- Therefore, the use of points can cause the interaction matrix to reach a singular configuration during visual control
- Example of singularity: Consider a desired camera movement of 180 degrees around the Z-axis. The camera moves towards infinity and the interaction matrix reduces its range from 6 to 2.



Index

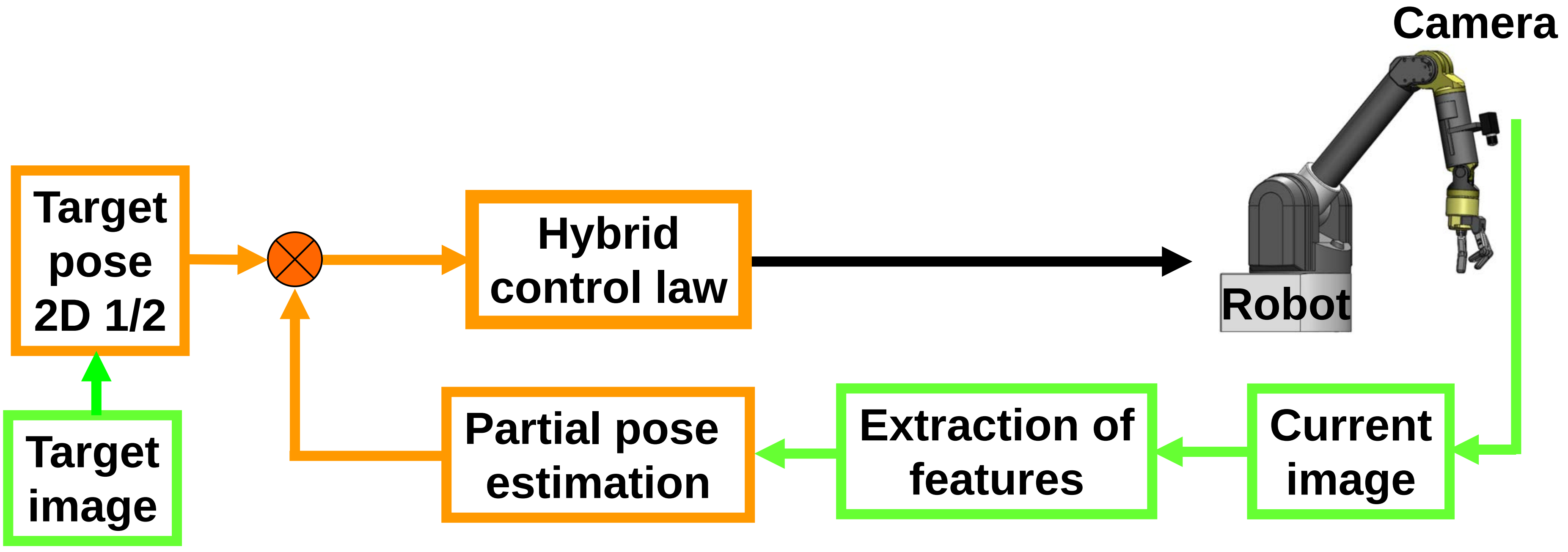
- Introduction
- Robotic manipulation
- Vision systems
- Classification of visual control systems
- Position-Based Visual Servoing
- Image-Based Visual Servoing
- Stability analysis

- **Hybrid Visual Servoing**
  - Hybrid visual servoing 2-1/2D

- Conclusion and bibliography

Hybrid visual servoing

- Hybrid visual servoing
  - Partially based on position and image
  - Partial decoupling in the interaction matrix
  - Presents the advantages of both position-based and image-based methods while overcoming their drawbacks
  - Combines 2D and 3D information



Hybrid visual servoing 2-1/2D

- 2-1/2D visual servoing
  - Method that avoids the drawbacks of classical methods:
    - Unlike position-based methods it does not require a 3D model of the object.
    - Unlike image-based methods, convergence is guaranteed over the entire workspace.
  - This control decouples rotational and translational motions by appropriate selection of visual features partially defined in 2D and 3D.
    - Partial estimation of camera motion
    - Robustness against calibration errors
    - Asymptotic stability guarantees

  - E. Malis, F. Chaumette, and S. Boudet, "2-1/2D visual servoing," *IEEE Trans. Robot. Automat.*, vol. 15, pp. 238-250, Apr. 1999

Hybrid visual servoing 2-1/2D

- 2-1/2D Visual servoing: Image features $\dot{\mathbf{s}} = \mathbf{L_h} \cdot \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix}$
  - We select the following image features

$$\mathbf{s} = \begin{pmatrix} \mathrm{x} & \mathrm{y} & \log\mathrm{Z} & \theta \cdot \mathbf{u} \end{pmatrix}, \text{ with } \mathbf{u} = \begin{pmatrix} u_x & u_y & u_z \end{pmatrix}^T$$

$$\mathbf{s}^* = \begin{pmatrix} \mathrm{x}^* & \mathrm{y}^* & \log\mathrm{Z}^* & \mathbf{0} \end{pmatrix}$$

  - Where θ and $\mathbf{u}$ are the angle and axis defining the rotation, so that the orientation control is based on position.
  - We define the error vector as

$$\mathbf{e} = \mathbf{s} - \mathbf{s}^* = \begin{pmatrix} \mathrm{x} - \mathrm{x}^* & \mathrm{y} - \mathrm{y}^* & \log \rho_\mathrm{z} & \theta \cdot \mathbf{u} \end{pmatrix}^\mathrm{T}, \text{ with } \rho_\mathrm{z} = \left(\mathrm{Z} / \mathrm{Z}^* \right)$$

  - We want an exponential decrease of the error

$$\dot{\mathbf{e}} = -\lambda \cdot \mathbf{e} = \dot{\mathbf{s}}$$

$$-\lambda \cdot \mathbf{e} = \dot{\mathbf{s}} = \mathbf{L_h} \cdot \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix} \Rightarrow \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix} = -\lambda \cdot \mathbf{L_h^+} \cdot \mathbf{e}$$

Hybrid visual servoing 2-1/2D

- 2-1/2D Visual servoing: Interaction Matrix
  - The hybrid interaction matrix is defined:

$$\dot{\mathbf{s}} = \mathbf{L_h} \cdot \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix} = \begin{pmatrix} \mathbf{L_v} & \mathbf{L_\omega} \\ \mathbf{0} & \mathbf{L_{\theta u}} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix}$$

$$\mathbf{L_v} = \frac{1}{Z^* \cdot \rho_z} \begin{pmatrix} -1 & 0 & x \\ 0 & -1 & y \\ 0 & 0 & -1 \end{pmatrix} \qquad \mathbf{L}_\omega = \begin{pmatrix} x \cdot y & -(1+x^2) & y \\ 1+y^2 & -x \cdot y & -x \\ -y & x & 0 \end{pmatrix}$$

$$\mathbf{L_{\theta u}} = \mathbf{I}_3 - \frac{\theta}{2} \cdot [\mathbf{u}]_\times + \left(1 - \frac{\mathrm{sinc}(\theta)}{\mathrm{sinc}^2(\theta/2)}\right) \cdot [\mathbf{u}]_\times^2$$

  - **Z*** must be estimated.
  - By selecting a point in the image (**x, y**) we have that the matrix **Lv** is triangular and always invertible
  - The ratio **Z/Z*** can be calculated using the homography

Hybrid visual servoing 2-1/2D

- 2-1/2D visual servoing: Control law
  - Computation of control velocities

$$-\lambda \cdot \mathbf{e} = \dot{\mathbf{s}} = \begin{pmatrix} \mathbf{L_v} & \mathbf{L_\omega} \\ \mathbf{0} & \mathbf{L_{\theta u}} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix} \Rightarrow \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix} = -\lambda \cdot \begin{pmatrix} \mathbf{L_v^+} & -\mathbf{L_v^+} \cdot \mathbf{L_\omega} \cdot \mathbf{L_{\theta u}^{-1}} \\ \mathbf{0} & \mathbf{L_{\theta u}^{-1}} \end{pmatrix} \cdot \mathbf{e}$$

$$\mathbf{e} = \left(\mathbf{e_t} \quad \mathbf{e_\omega}\right)^T = \left(x - x^* \quad y - y^* \quad \log \rho_z \quad \theta \cdot \mathbf{u}\right)^T \rightarrow \mathbf{e_\omega} = \theta \cdot \mathbf{u}$$

  - Rotation velocities are based on position:

$$\boldsymbol{\omega} = -\lambda \cdot \mathbf{L_{\theta u}^{-1}} \cdot \mathbf{e_\omega} = -\lambda \cdot \mathbf{L_{\theta u}^{-1}} \cdot \theta \cdot \mathbf{u} \text{ , with } \mathbf{L_{\theta u}} \cdot \theta \cdot \mathbf{u} = \mathbf{L_{\theta u}^{-1}} \cdot \theta \cdot \mathbf{u} = \theta \cdot \mathbf{u}$$

$$\boxed{\boldsymbol{\omega} = -\lambda \cdot \theta \cdot \mathbf{u}}$$

  - Translation velocities are image-based:

$$\mathbf{v} = -\lambda \cdot \mathbf{L_v^+} \cdot \left(\mathbf{e_t} - \mathbf{L_\omega} \cdot \mathbf{L_{\theta u}^{-1}} \cdot \mathbf{e_\omega}\right) = -\mathbf{L_v^+} \cdot \left(\lambda \cdot \mathbf{e_t} - \lambda \cdot \mathbf{L_\omega} \cdot \theta \cdot \mathbf{u}\right)$$

$$\boxed{\mathbf{v} = -\mathbf{L_v^+} \cdot \left(\lambda \cdot \mathbf{e_t} + \mathbf{L_\omega} \cdot \boldsymbol{\omega}\right)}$$

    - The computed velocity depends on the translation part of the error and the angular velocity of the control

Index

- Introduction
- Robotic manipulation
- Vision systems
- Classification of visual control systems
- Position-Based Visual Servoing
- Image-Based Visual Servoing
- Stability analysis
- Hybrid Visual Servoing

- **Conclusion and bibliography**

Conclusion

- Visual servoing / Visual control
  - Controlling the position of a robot using vision information allows the positioning of the robot terminal element in an unstructured environment.

- Position-based visual servoing
  - Allows task planning in Cartesian space
  - 3D reconstruction is required
  - Sensitive to calibration errors
  - No control in the image. Object of interest may leave the field of view

- Image-based visual servoing
  - No explicit reconstruction needed. Eliminates geometrical model errors
  - Robustness with respect to camera and robot calibration errors
  - Local minima or singularities of the image Jacobian
  - No direct control in Cartesian space. Can produce elegant trajectories in the image but twisted in Cartesian space

- Hybrid visual servoing
  - Decoupling of degrees of freedom by combining control techniques

Conclusion

- Visual control challenges
  - One feature for each robot degree of freedom
  - Perfect decoupling between image features and robot degrees of freedom
  - Global stability: No singularities or local minima
  - Transformation to a linear system control

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