No existen cargas ni corrientes libres:

$$\nabla \cdot \mathbf{E}(\mathbf{r},t) = 0$$
$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t)$$
$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0$$
$$\nabla \times \mathbf{B}(\mathbf{r},t) = \mu\sigma\mathbf{E}(\mathbf{r},t) + \mu\varepsilon\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r},t)$$

Se toma el rotacional de la ley de Faraday

$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial(\nabla \times \mathbf{B})}{\partial t}$$

...

Ecuación de ondas:

$$\nabla^2\mathbf{E} - \mu\sigma\frac{\partial\mathbf{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$$
$$\nabla^2\mathbf{B} - \mu\sigma\frac{\partial\mathbf{B}}{\partial t} - \mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial t^2} = 0$$

$$\nabla^2\psi - \mu\sigma\frac{\partial\psi}{\partial t} - \mu\varepsilon\frac{\partial^2\psi}{\partial t^2} = 0$$

donde ψ es cada una de las componentes del campo eléctrico o magnético

$$\nabla^2 \psi - \mu\varepsilon \frac{\partial^2 \psi}{\partial t^2} = 0$$

La luz es una onda EM:

$\psi(x,t) = f(x - vt) + g(x + vt)$ es una solución genérica de las ecuaciones de onda

Para $f(x - vt), w = x - vt \rightarrow (1 - v^2 \mu\varepsilon) \dfrac{\partial^2 f}{\partial w^2} = 0$

Y por lo tanto:

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

donde:

- ψ es una de las componentes del campo eléctrico o magnético
- Igual con $g(x + vt)$
- $v = c/n$ y $n = \sqrt{\varepsilon_r \mu_r}$

This wave consists entirely of magnetic disturbances, the direction of magnetization being in the plane of the wave. No magnetic disturbance whose direction of magnetization is not in the plane of the wave can be propagated as a plane wave at all.

Hence magnetic disturbances propagated through the electromagnetic field agree with light in this, that the disturbance at any point is transverse to the direction of propagation, and such waves may have all the properties of polarized light.

(96) The only medium in which experiments have been made to determine the value of k is air, in which $\mu=1$, and therefore, by equation (46),

$$V=v. \quad \quad (72)$$

By the electromagnetic experiments of MM. WEBER and KOHLRAUSCH*,

$$v=310,740,000 \text{ metres per second}$$

is the number of electrostatic units in one electromagnetic unit of electricity, and this, according to our result, should be equal to the velocity of light in air or vacuum.

The velocity of light in air, by M. FIZEAU'S† experiments, is

$$V=314,858,000;$$

according to the more accurate experiments of M. FOUCAULT‡,

$$V=298,000,000.$$

The velocity of light in the space surrounding the earth, deduced from the coefficient of aberration and the received value of the radius of the earth's orbit, is

$$V=308,000,000.$$

(97) Hence the velocity of light deduced from experiment agrees sufficiently well with the value of v deduced from the only set of experiments we as yet possess. The value of v was determined by measuring the electromotive force with which a condenser of known capacity was charged, and then discharging the condenser through a galvanometer, so as to measure the quantity of electricity in it in electromagnetic measure. The only use made of light in the experiment was to see the instruments. The value of V found by M. FOUCAULT was obtained by determining the angle through which a revolving mirror turned, while the light reflected from it went and returned along a measured course. No use whatever was made of electricity or magnetism.

The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

(98) Let us now go back upon the equations in (94), in which the quantities J and Ψ occur, to see whether any other kind of disturbance can be propagated through the medium depending on these quantities which disappeared from the final equations.

* Leipzig Transactions, vol. v. (1857), p. 260, or POGGENDORFF'S 'Annalen,' Aug. 1856, p. 10.

† Comptes Rendus, vol. xxix. (1849), p. 90.

‡ Ibid. vol. lv. (1862), pp. 501, 792.

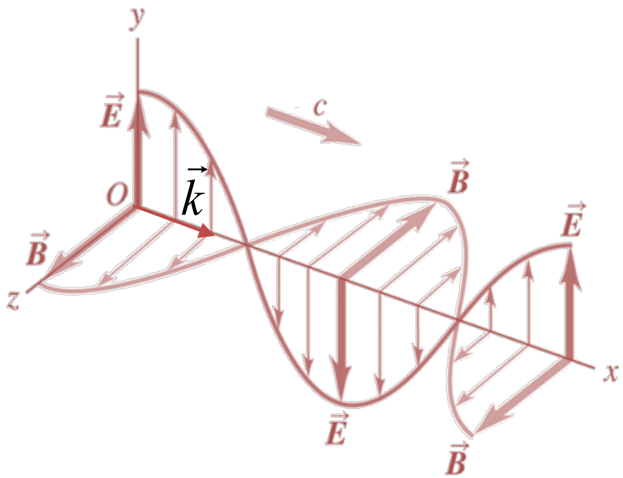
"*A Dynamical theory of the electromagnetic field*", Phil. Trans. R Soc. Lond. 155, 459-512 (1865).
https://royalsocietypublishing.org/doi/pdf/10.1098/rstl.1865.0008

Si $\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$ y $\psi(x,t) = X(x)T(t) \Rightarrow$

$$\frac{d^2X}{dx^2} + k^2X = 0$$

$$\frac{d^2T}{dt^2} + \omega^2T = 0$$

donde $k^2 = \frac{\omega^2}{v^2}$ es la relación de dispersión



...

Para una onda viajera en la dirección x se llega a que:

$$\mathbf{E} = \mathbf{E}_0 e^{\imath(kx-\omega t)}$$

$$\mathbf{B} = \mathbf{B}_0 e^{\imath(kx-\omega t)}$$

Y para una dirección cualquiera:

$$\mathbf{E} = \mathbf{E}_0 e^{\imath(\mathbf{kr}-\omega t)}$$

$$\mathbf{B} = \mathbf{B}_0 e^{\imath(\mathbf{kr}-\omega t)}$$

siendo $\mathbf{k} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}$

Las ecuaciones de Maxwell

$$\nabla\cdot\mathbf{E}(\mathbf{r},t) = 0$$
$$\nabla\times\mathbf{E}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t)$$
$$\nabla\cdot\mathbf{B}(\mathbf{r},t) = 0$$
$$\nabla\times\mathbf{B}(\mathbf{r},t) = \mu\varepsilon\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r},t)$$

se pueden escribir entonces

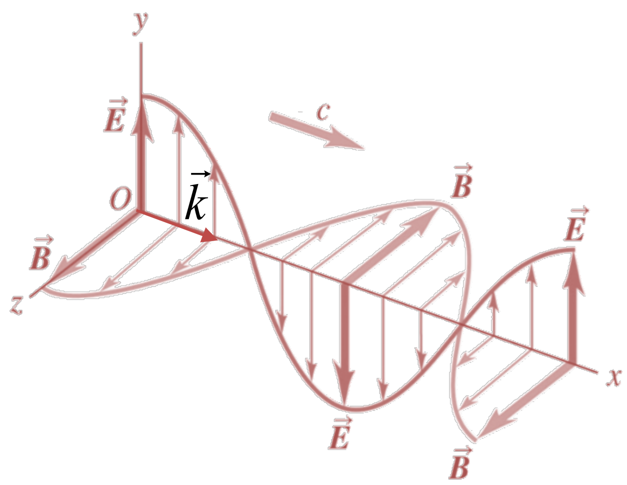
$$\mathbf{k}\cdot\mathbf{E} = 0$$
$$\mathbf{k}\times\mathbf{E} = \omega\mathbf{B}$$
$$\mathbf{k}\cdot\mathbf{B} = 0$$
$$\mathbf{k}\times\mathbf{B} = -\omega\mu\varepsilon\mathbf{E}$$

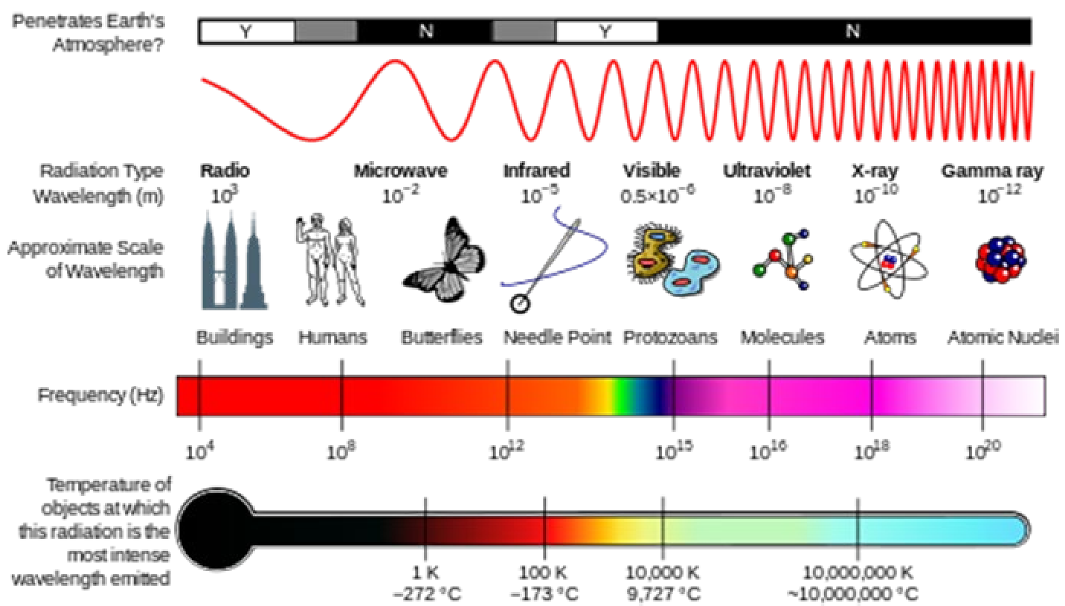
y por lo tanto

$$\mathbf{B} = \frac{k}{\omega}\hat{\mathbf{k}}\times\mathbf{E}$$

Class			Wave-length λ	Freq-uency f	Energy per photon E
Ionizing radiation	γ	Gamma rays	1 pm	300 EHz	1.24 MeV
	HX	Hard X-rays	10 pm	30 EHz	124 keV
	SX	Soft X-rays	100 pm	3 EHz	12.4 keV
			1 nm	300 PHz	1.24 keV
	EUV	Extreme ultraviolet	10 nm	30 PHz	124 eV
			100 nm	3 PHz	12.4 eV
Visible	NUV	Near ultraviolet, visible	1 μm	300 THz	1.24 eV
	NIR	Near infrared	10 μm	30 THz	124 meV
	MIR	Mid infrared	100 μm	3 THz	12.4 meV
	FIR	Far infrared	1 mm	300 GHz	1.24 meV
Micro-waves	EHF	Extremely high frequency	1 cm	30 GHz	124 μeV
	SHF	Super high frequency	1 dm	3 GHz	12.4 μeV
	UHF	Ultra high frequency	1 m	300 MHz	1.24 μeV
radio waves	VHF	Very high frequency	10 m	30 MHz	124 neV
	HF	High frequency	100 m	3 MHz	12.4 neV
	MF	Medium frequency	1 km	300 kHz	1.24 neV
	LF	Low frequency	10 km	30 kHz	124 peV
	VLF	Very low frequency	100 km	3 kHz	12.4 peV
	ULF	Ultra low frequency	1 Mm	300 Hz	1.24 peV
	SLF	Super low frequency	10 Mm	30 Hz	124 feV
	ELF	Extremely low frequency	100 Mm	3 Hz	12.4 feV

Sources: File:Light spectrum.svg[1][2][3]





En el campo lejano, es decir, lejos de las fuentes y los límites materiales, el campo electromagnético puede aproximarse localmente mediante una onda plana. Los campos eléctrico y magnético están en fase, perpendiculares entre sí, y la relación de sus amplitudes es constante.

Entonces $\langle \mathbf{S} \rangle$ se puede expresar por el campo eléctrico solo como

$$\langle \mathbf{S}(\mathbf{r}) \rangle = \frac{1}{2}\frac{1}{Z_i}|\mathbf{E}(\mathbf{r})|^2 \mathbf{n}_r = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}n_i|\mathbf{E}(\mathbf{r})|^2$$

donde $\mathbf{n}_r$ representa el vector unitario en la dirección radial, $n_i = \sqrt{\varepsilon_i \mu_i}$ es el índice de refracción, y Z_i es la impedancia.

La integral de superficie $\langle \mathbf{S} \rangle$ se corresponde con la potencia total generada o disipada en el interior de la superficie cerrada, esto es,

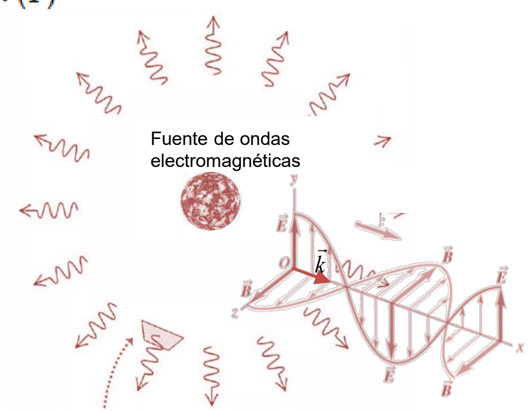
$$\bar{P} = \oint_A \langle \mathbf{S}(\mathbf{r}) \rangle \cdot \mathbf{n} da = \oint_A I(\mathbf{r}) da$$

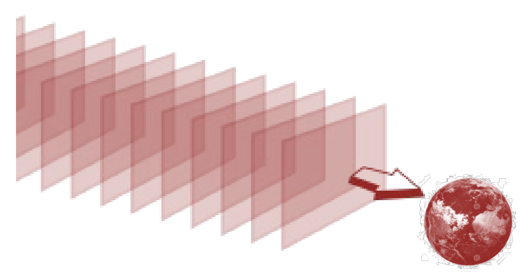
Se puede demostrar también que:

$$\langle \mathbf{S}(\mathbf{r}) \rangle = \langle u(\mathbf{r}) \rangle \mathbf{v}(\mathbf{r})$$

donde:

- $\langle u(\mathbf{r}) \rangle$ es el promedio de la densidad total de energía.
- $\mathbf{v}(\mathbf{r})$ la velocidad de fase en el medio.





$$\nabla^2\psi - \mu\sigma\frac{\partial\psi}{\partial t} - \mu\varepsilon\frac{\partial^2\psi}{\partial t^2} = 0$$

ψ es cada una de las componentes del campo eléctrico o magnético

Para $\psi = \psi_0 e^{\imath(kx-\omega t)}$ encontramos que $k^2 = \omega^2\mu\varepsilon + \imath\omega\mu\sigma$

Si se toma $k = \pm(k_r + \imath k_i) \Rightarrow \psi = \psi_0 e^{-k_i x} e^{\imath(k_r x - \omega t)}$

$$k_r = \omega\sqrt{\frac{\mu\varepsilon}{2}}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2}+1]^{1/2}$$
$$k_i = \omega\sqrt{\frac{\mu\varepsilon}{2}}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2}-1]^{1/2}$$

Discusión sobre:

- Longitud de onda $\lambda = 2\pi/k_r$
- Profundidad de penetración $\delta = 1/k_i$
- Índice de refracción $n = n_r + \imath n_i = ck/\omega$
- Velocidad en el medio $v = c/n$
- Desfase entre $\mathbf{B}$ y $\mathbf{E}$
- Definición del factor de calidad $Q = \dfrac{\omega\varepsilon}{\sigma}$ e interpretación física $Q = \dfrac{|\frac{\partial}{\partial t}\mathbf{D}|}{|\mathbf{J}_{\mathbf{cond}}|}$
- Distintos regímenes según el factor de calidad $Q \rightarrow$ Aislante vs Conductor.

Las ecuaciones de Maxwell

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$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0$$
$$\nabla \times \mathbf{B}(\mathbf{r},t) = \mu\sigma\mathbf{E}(\mathbf{r},t) + \mu\varepsilon\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r},t)$$

se pueden escribir entonces

$$\mathbf{k}\cdot\mathbf{E} = 0$$
$$\mathbf{k}\times\mathbf{E} = \omega\mathbf{B}$$
$$\mathbf{k}\cdot\mathbf{B} = 0$$
$$\mathbf{k}\times\mathbf{B} = -(\imath\mu\sigma + \omega\mu\varepsilon)\mathbf{E}$$

y por lo tanto $\mathbf{B} = \dfrac{k}{\omega}\hat{\mathbf{k}} \times \mathbf{E}$

Modelo microscópico de la conductividad.

$$\mathbf{F} = m\mathbf{a} = -e\mathbf{E} - \gamma\mathbf{v}$$

Para un campo estático la conductividad resulta ser:

$$\sigma_0 = \frac{ne^2}{\gamma}$$

Para la excitación mediante una OEM de la forma $\mathbf{E} = \mathbf{E}_0 e^{\imath(\mathbf{kr}-\omega t)} = \mathbf{E}(\mathbf{r})e^{-\imath\omega t}$ y teniendo en cuenta que:

- $\mathbf{v} = \mathbf{v}_0 e^{-\imath\omega t}$
- $\mathbf{J} = \sigma\mathbf{E}$
- $\mathbf{J} = -ne\mathbf{v}$

Entonces:

$$\sigma(\omega) = \sigma_R + \imath\sigma_I = \frac{ne^2}{\gamma - \imath m\omega} = \frac{ne^2/\gamma}{1 - \imath m\omega/\gamma} = \frac{\sigma_0}{1 - \imath(\sigma_0 m\omega/ne^2)}$$

Esta conductividad da lugar a una relación de dispersión:

$$k^2 = \omega^2\mu(\varepsilon + \imath\sigma(\omega)/\omega)$$

tal que

$$k_r = \omega\sqrt{\frac{\mu\varepsilon}{2}}[\sqrt{1 + (\frac{\sigma_R(\omega)}{\omega\varepsilon(\omega)})^2} + 1]^{1/2}$$

$$k_i = \omega\sqrt{\frac{\mu\varepsilon}{2}}[\sqrt{1 + (\frac{\sigma_R(\omega)}{\omega\varepsilon(\omega)})^2} - 1]^{1/2}$$

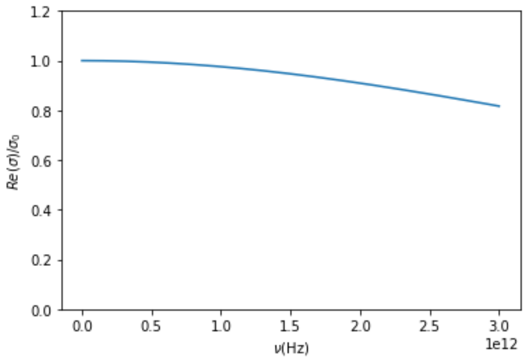
donde $\varepsilon(\omega) = \varepsilon - \frac{\sigma_I}{\omega}$

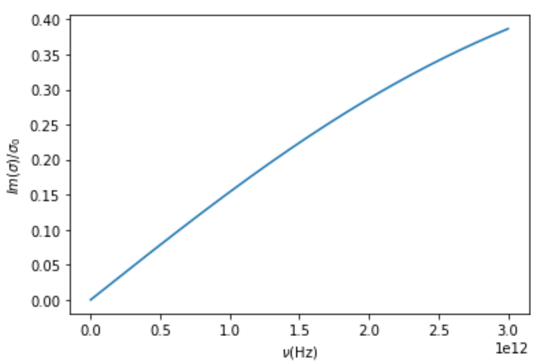
Límite $m\omega/\gamma \lll 1$ ó $\omega \lll ne^2/\sigma_0 m$

```
[24] from math import pi
     eps0=8.8541878176e-12 #F/m
     e=1.6e-19    # C
     m=9.11e-31   # kg
     #Cobre
     n=8.5e28     # m-3
     sigma0=6.0e7 #(ohm-metro)-1
     def sigma(omega):
       resigma=sigma0/(1+(sigma0*m*omega/(n*e**2))**2)
       imsigma=resigma*(sigma0*m*omega/(n*e**2))
       return resigma,imsigma
```

$$\sigma(\omega) = \sigma_R + \imath\sigma_I = \frac{\sigma_0}{1+(\sigma_0 m\omega/ne^2)^2}\left(1+\imath\frac{\sigma_0 m\omega}{ne^2}\right)$$

```
import numpy as np
import matplotlib.pyplot as plt
N = 1000

#Microondas
nuini=300.0e6 #Hz (Microondas, lambda=1m)
nufin=3.0e12  #Hz (Terahercios, lambda=100mu)

freq = np.linspace(nuini, nufin, N)
sigma_real,sigma_imag=sigma(2.0*pi*freq)

# Re(Sigma)
fig, ax  = plt.subplots()
ax.plot(freq,sigma_real/sigma0)
ax.set_xlabel(r'$\nu$(Hz)')
ax.set_ylabel("$Re(\sigma)/\sigma_0$")
ax.set_ylim(0,1.2)
plt.show()

# Im(Sigma)
fig, ax  = plt.subplots() #genera el objeto "figura"
ax.plot(freq,sigma_imag/sigma0)          #pinta la gráfica
ax.set_xlabel(r'$\nu$(Hz)')
ax.set_ylabel("$Im(\sigma)/\sigma_0$")
plt.show()
```





Límite $m\omega/\gamma \ggg 1$ ó $\omega \ggg ne^2/\sigma_0 m$ → $\sigma(\omega) = \sigma_R + \imath\sigma_I = \dfrac{\sigma_0}{1+(\sigma_0 m\omega/ne^2)^2}\left(1+\imath\dfrac{\sigma_0 m\omega}{ne^2}\right)$

$$\sigma(\omega) \approx \imath\left(\frac{ne^2}{m\omega}\right)$$

$$k^2 = \omega^2\mu\varepsilon\left(1-\frac{ne^2}{m\varepsilon\omega^2}\right) = \omega^2\mu\varepsilon_0\left(\varepsilon_r - \frac{\omega_p^2}{\omega^2}\right)$$

$\omega_p^2 = \dfrac{ne^2}{m\varepsilon_0}$ es la frecuencia de plasma.

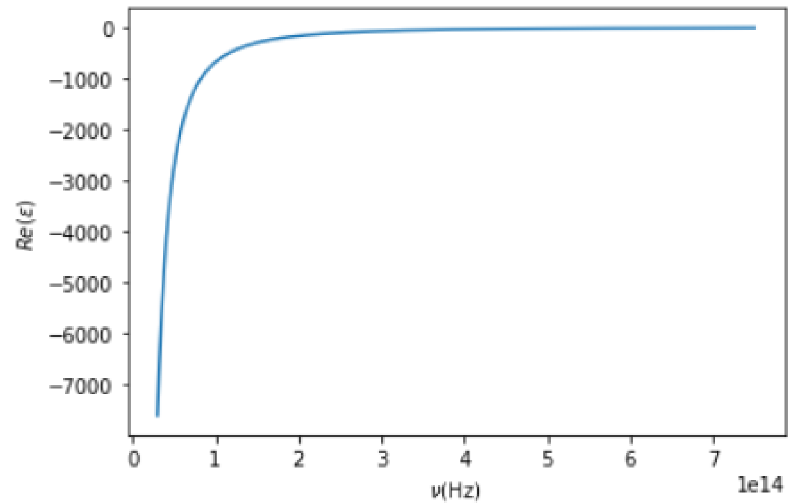
En materiales no magnéticos como los metales $\mu_r \approx 1$ y por lo tanto:

$$k^2 = \frac{\omega^2}{c^2}\left(\varepsilon_r - \frac{\omega_p^2}{\omega^2}\right)$$

Y por lo tanto el índice de refracción es $n^2 = \varepsilon(\omega) = \varepsilon_r - \dfrac{\omega_p^2}{\omega^2}$

```
def epsilon(omega):
  eps_r=1.0
  wp=np.sqrt(n*e**2/(m*eps0))
  return eps_r-(wp/omega)**2
```

```
Frecuencia de plasma (Hz) 1.6424641233724826e+16
Longitud de onda (nm) 114.7
```




```
#VISIBLE INFRARROJO
nuini=30.0e12 #Hz (INFRARROJO, lambda=1 micra)
nufin=750.0e12  #Hz (VISIBLE lambda=400 nm)
```